Causal Discovery from Interventions

Brady Neal

causalcourse.com

Structural Interventions

Single-Node Interventions

Multi-Node Interventions

Parametric Interventions

Interventional Markov Equivalence

Miscellaneous Other Settings

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Intervention in Two-Variable Setting

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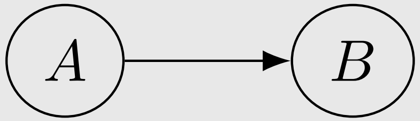

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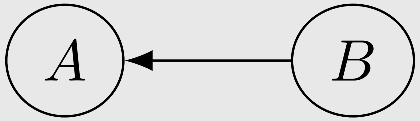
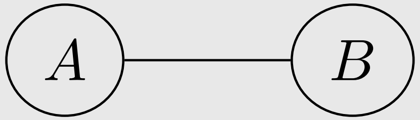


Same Markov equivalence class / essential graph



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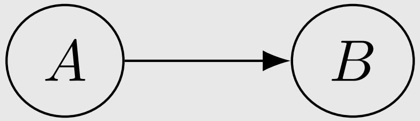

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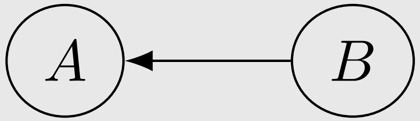

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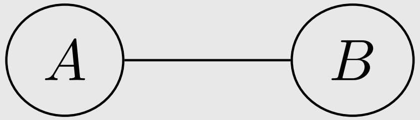

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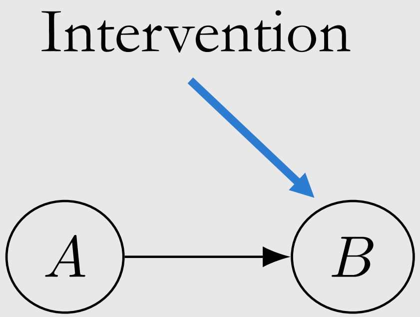

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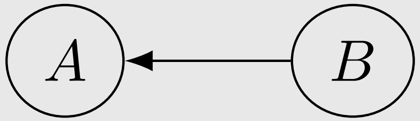

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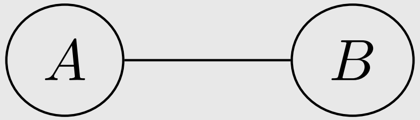

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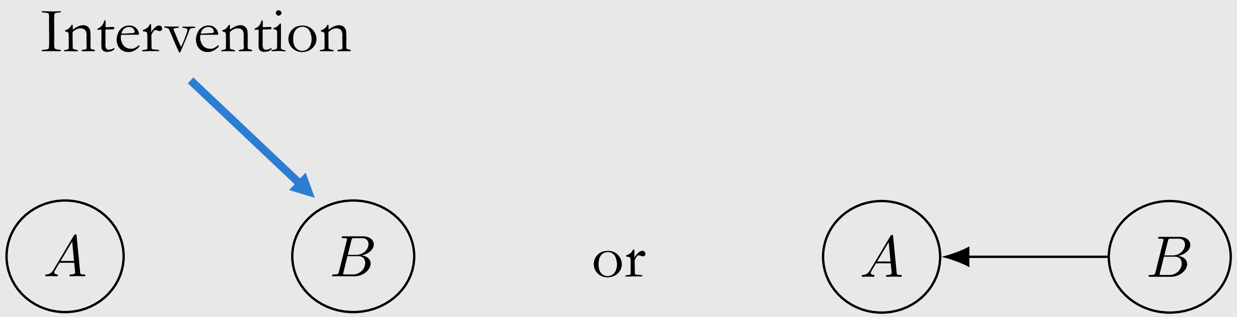

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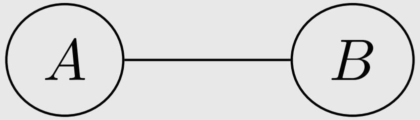

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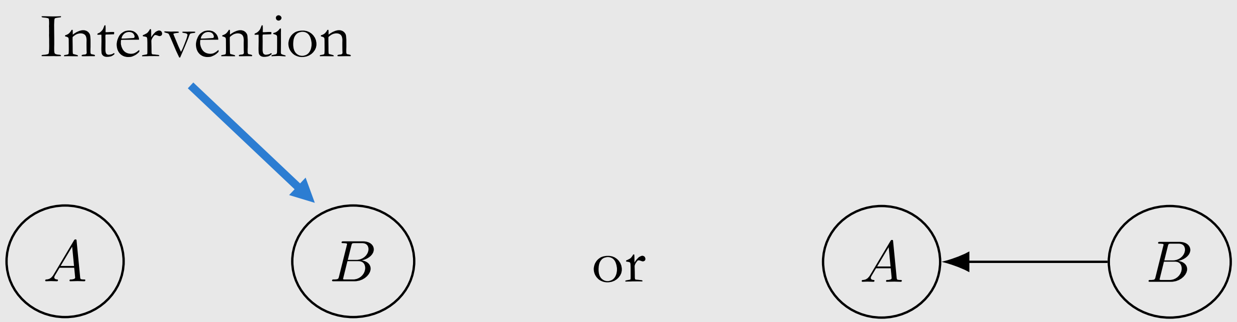
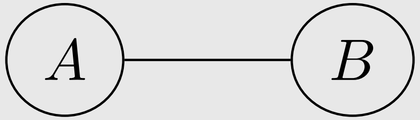


Manipulated graph

Same Markov equivalence class / essential graph



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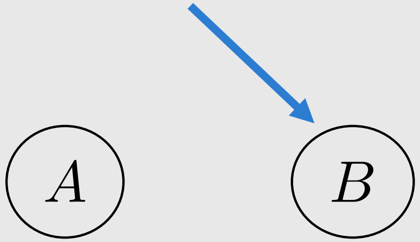
Intervention



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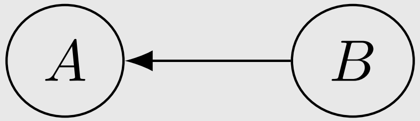

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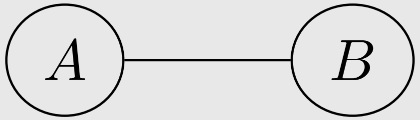

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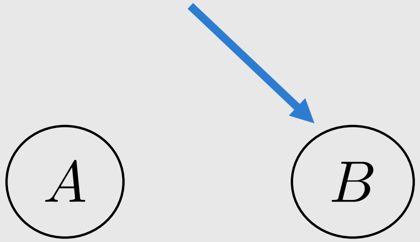

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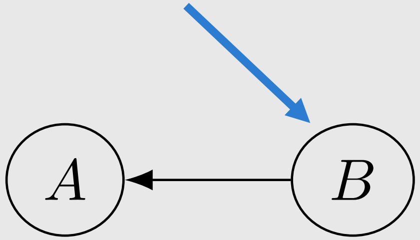

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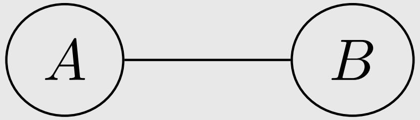

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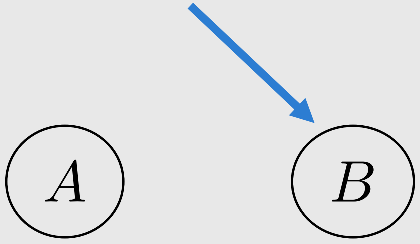

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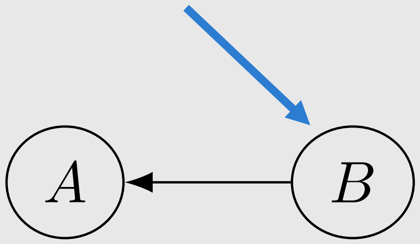

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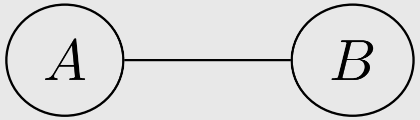

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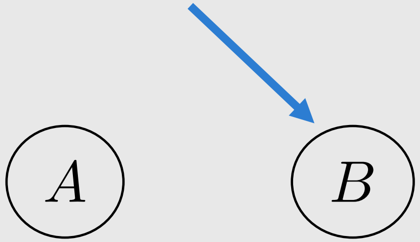

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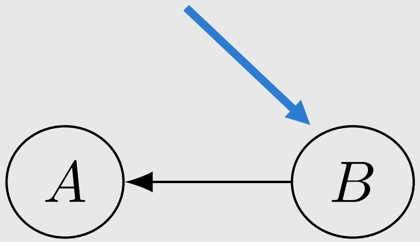

Manipulated graph

$B := N_B$

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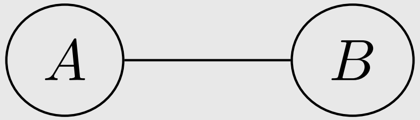

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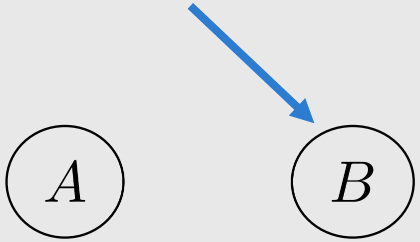

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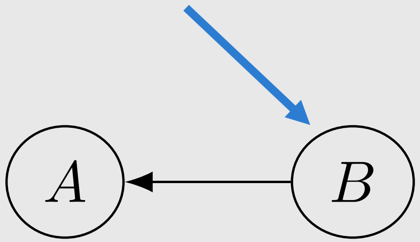

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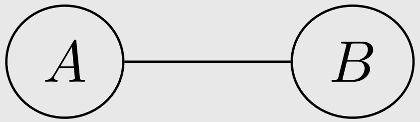
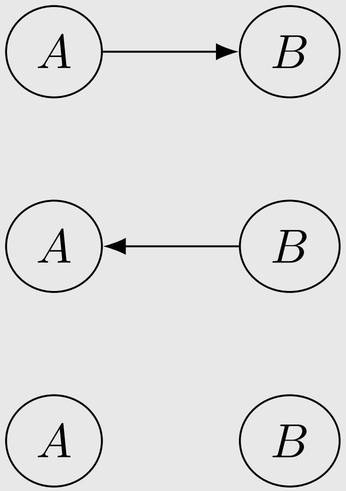


Two Variables: Interventional Essential Graphs

True graph



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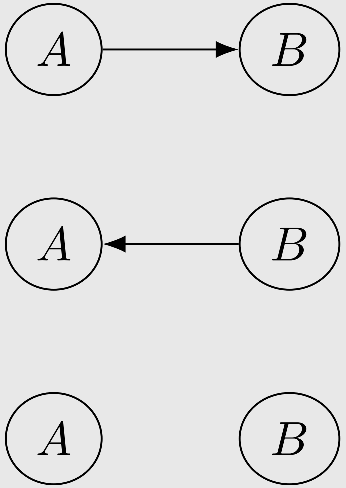

I = {A}

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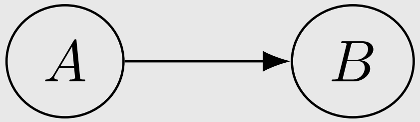

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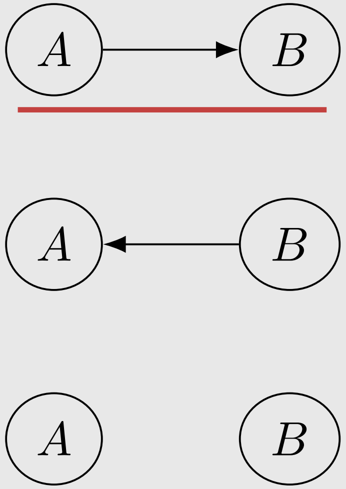

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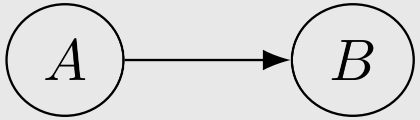

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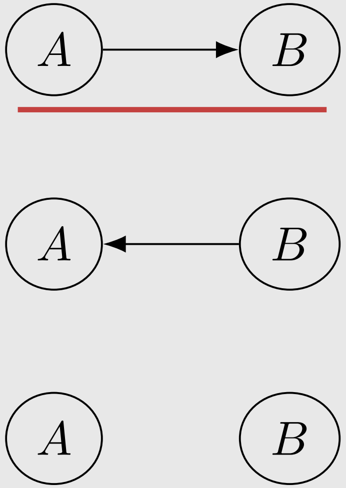

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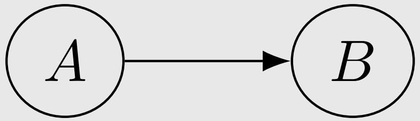

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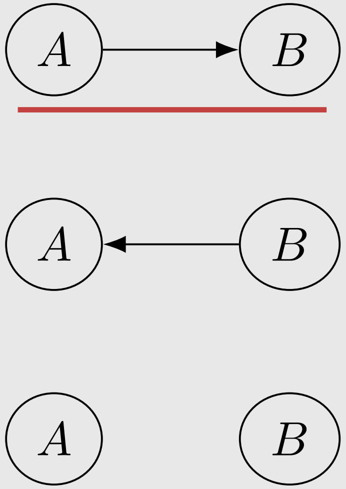

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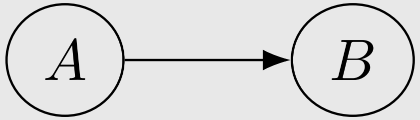

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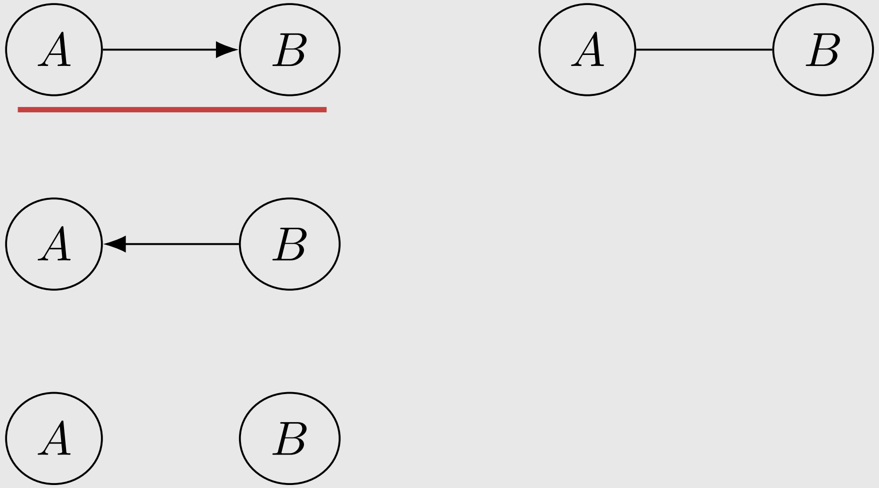

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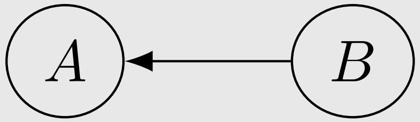

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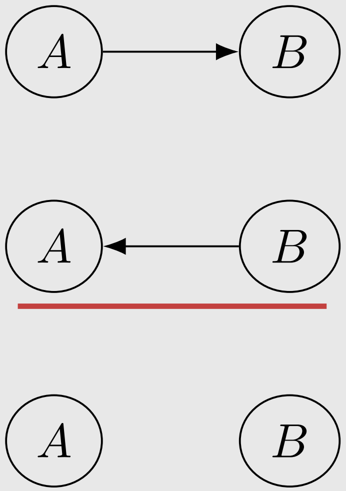

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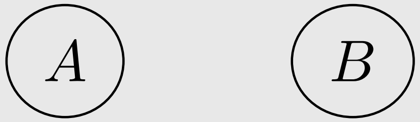

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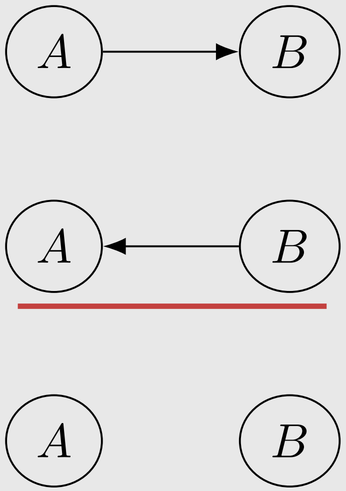

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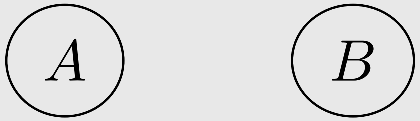

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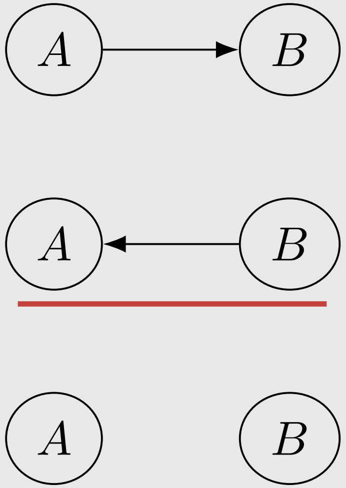

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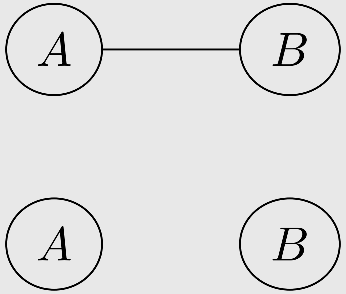

$I = \{B\}$

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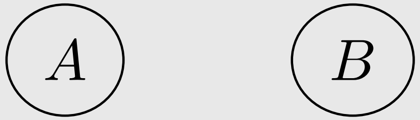

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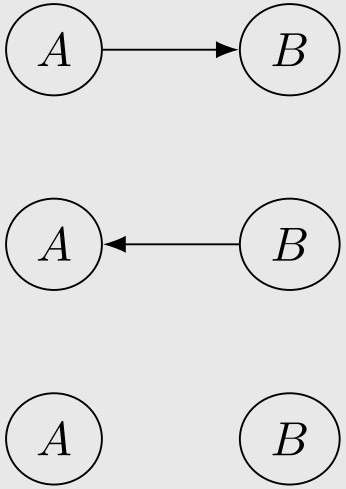

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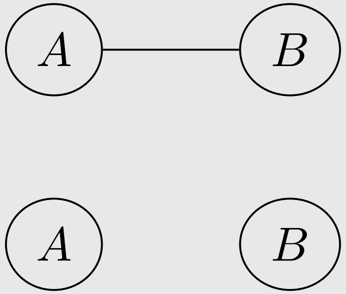

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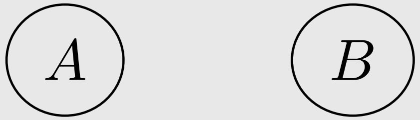

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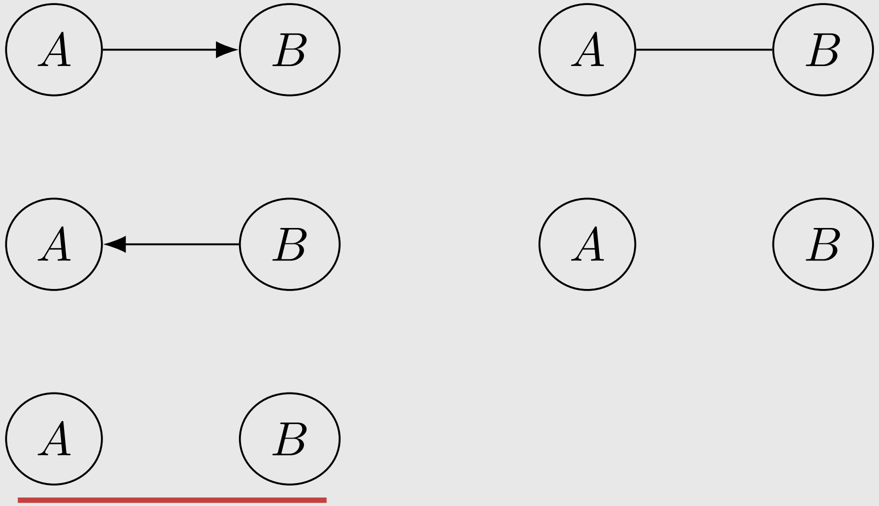

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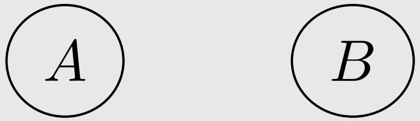

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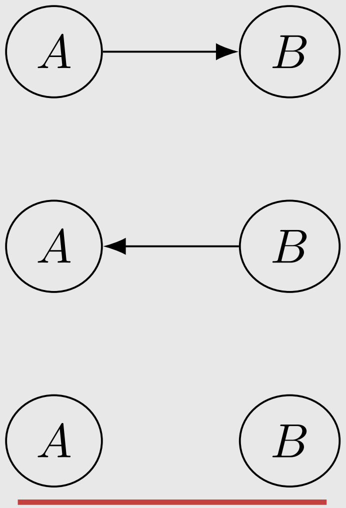

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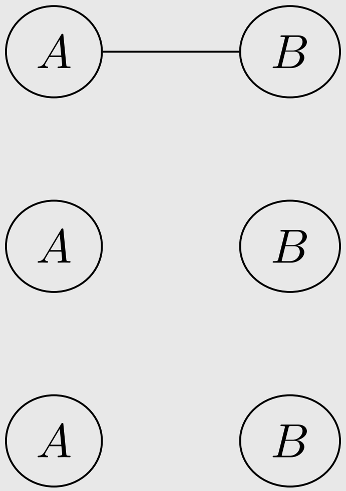

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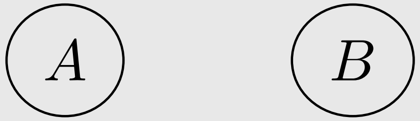

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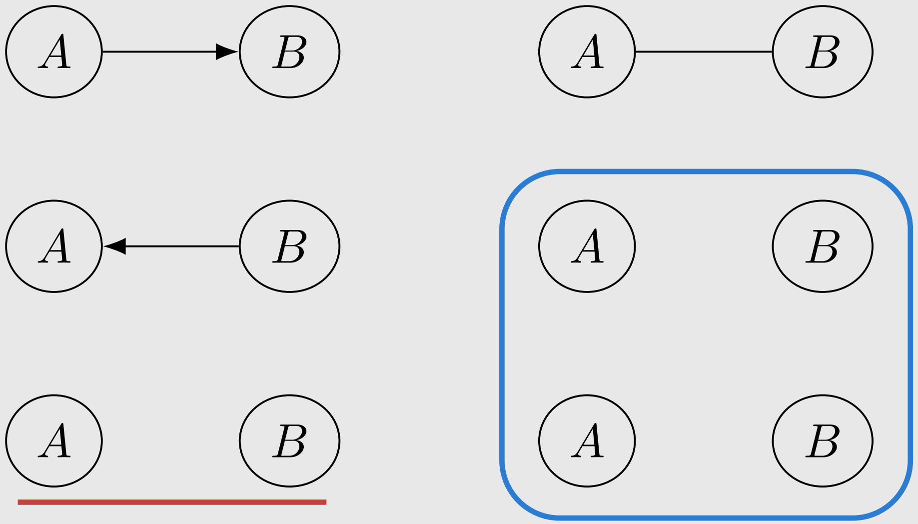

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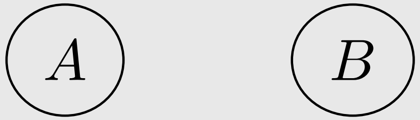
True graph under intervention I = {B} 

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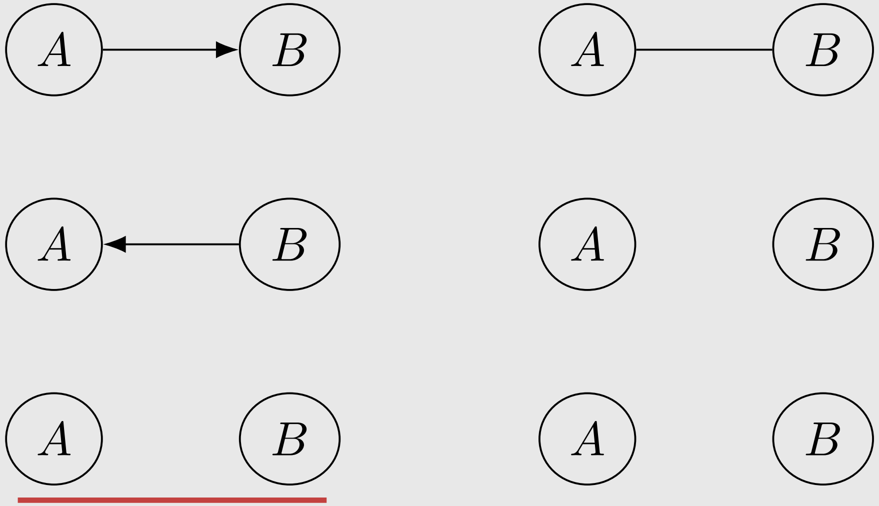

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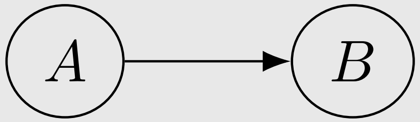

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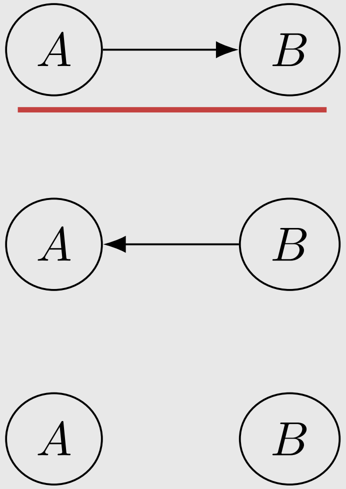

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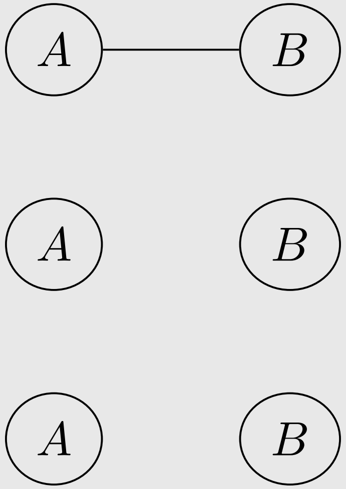

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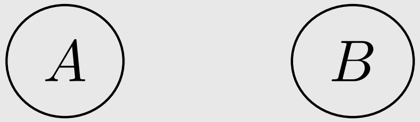

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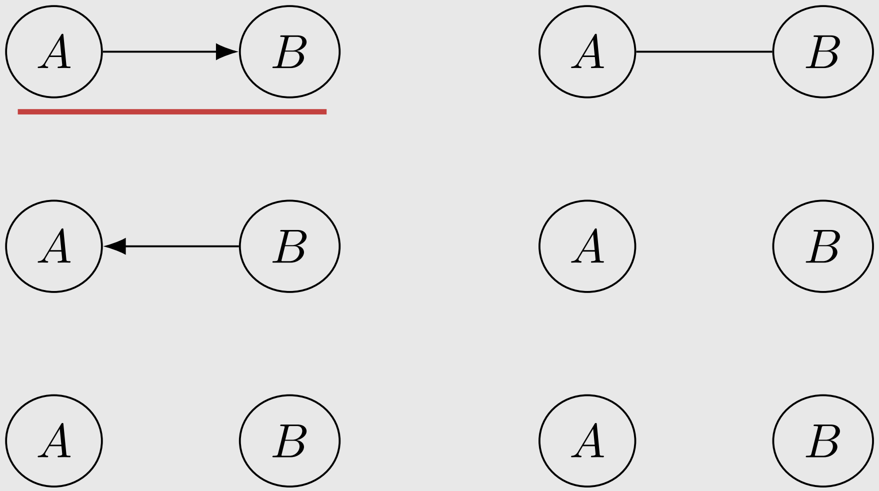

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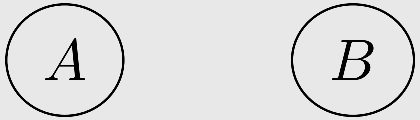

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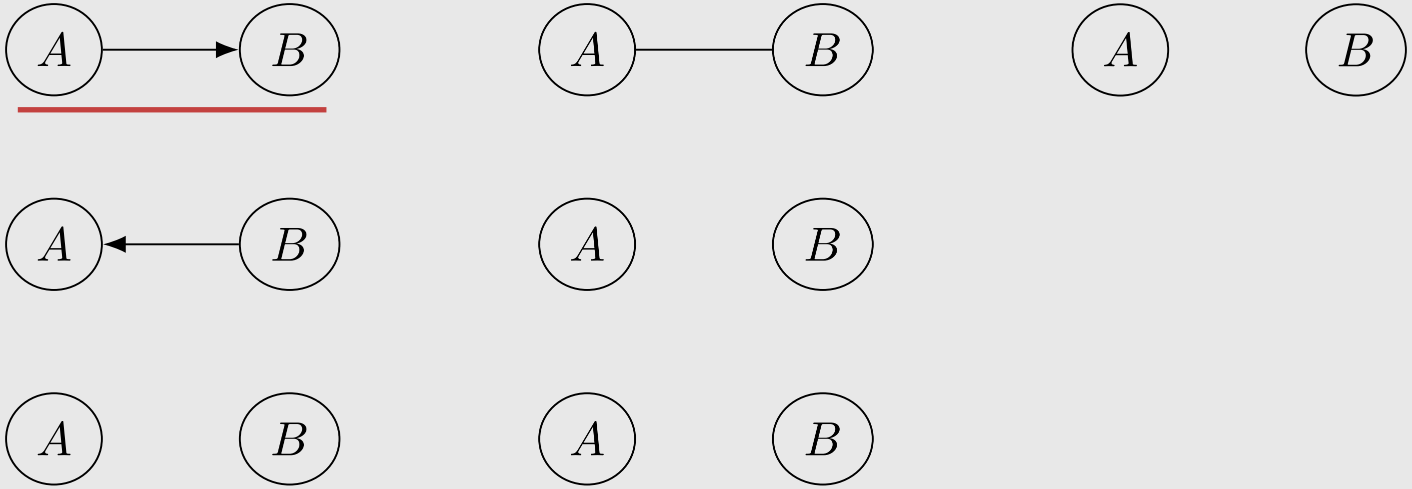

Two Variables: Interventional Essential Graphs

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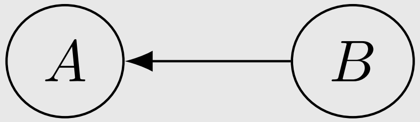
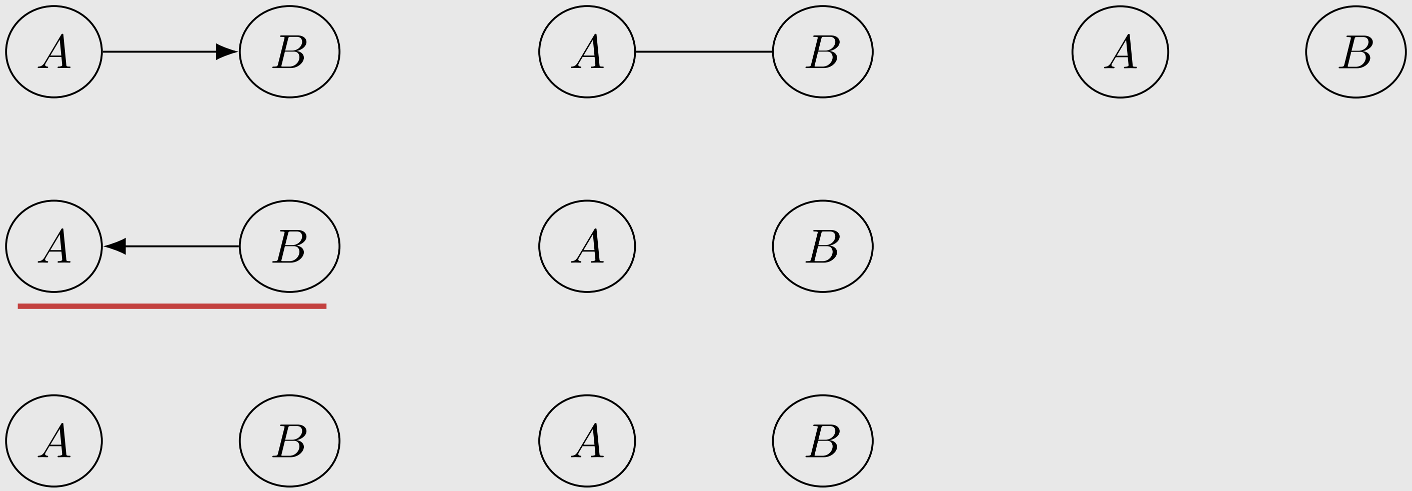


Interventional essential graphs

True graph	$I = \{A\}$	$I = \{B\}$	$I = \{\}$



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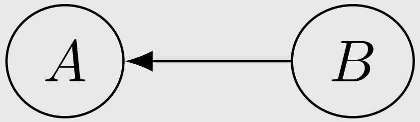

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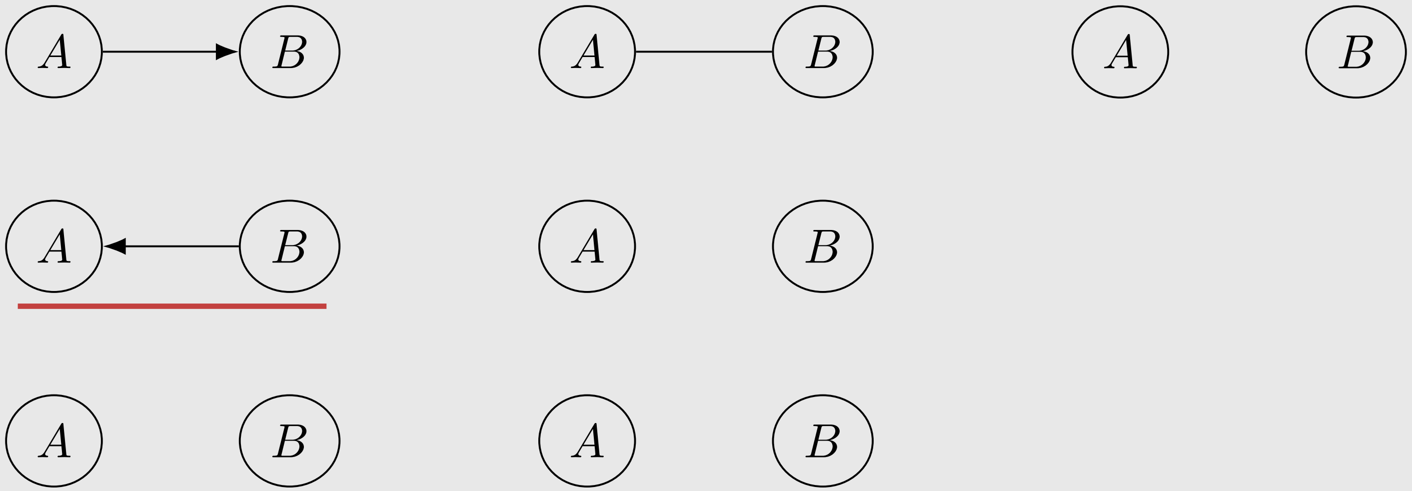
True graph | I = {A} | I = {B} | I = {}



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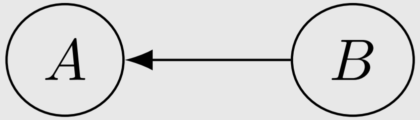

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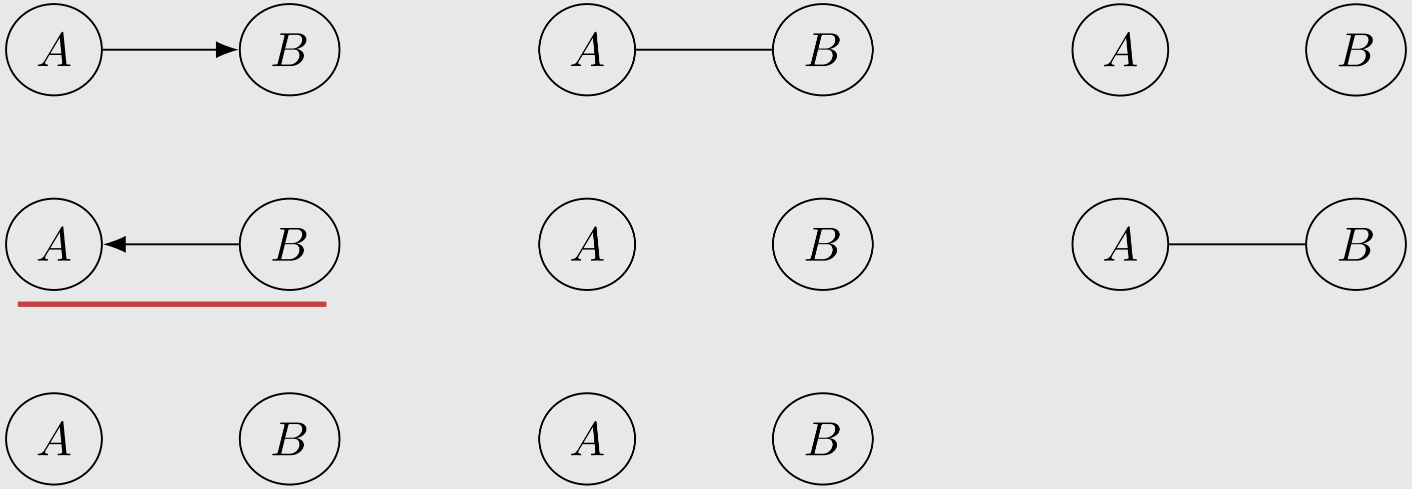

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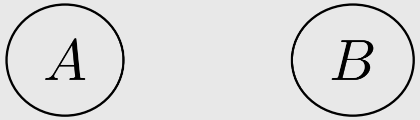

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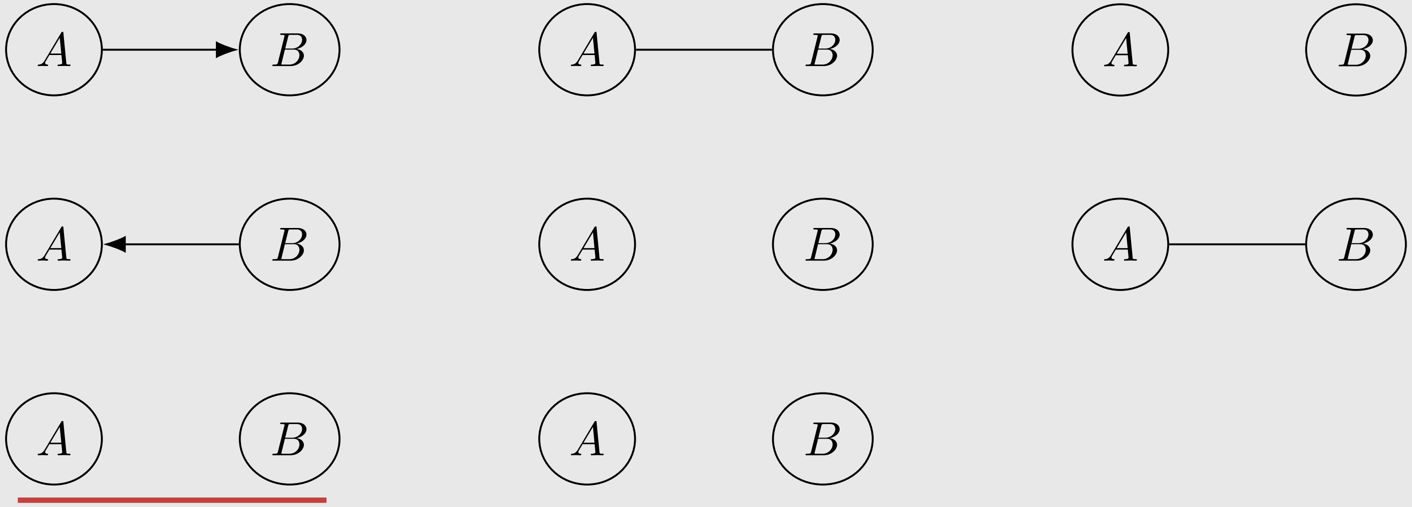

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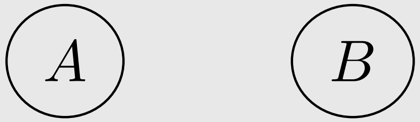

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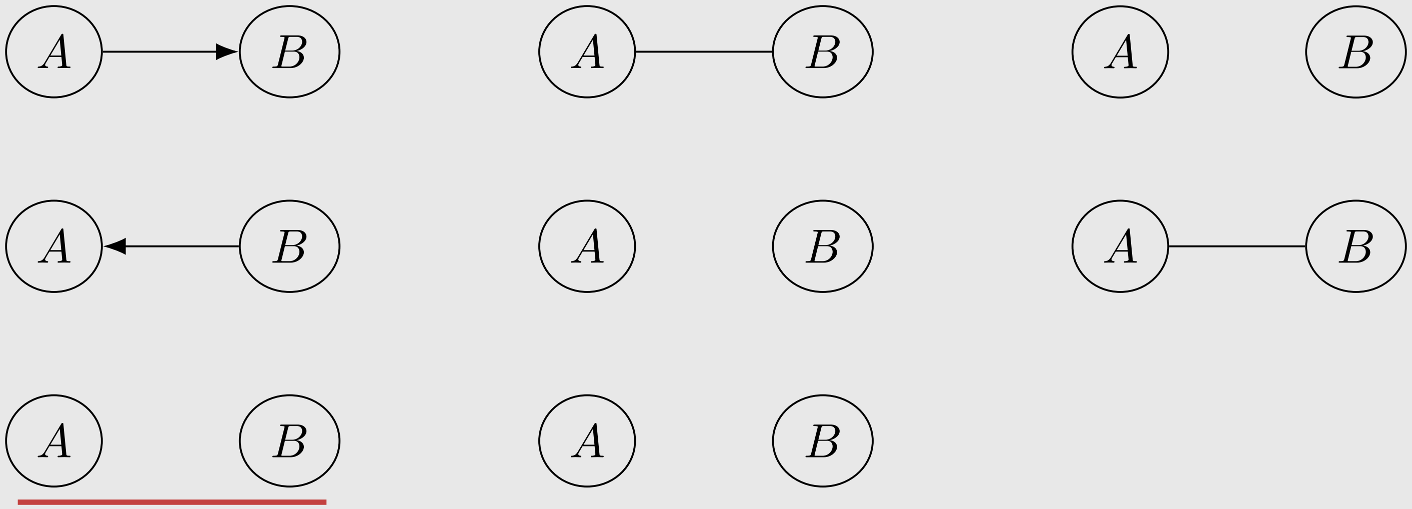

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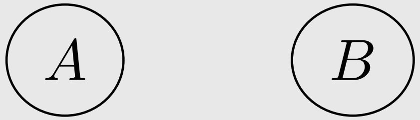

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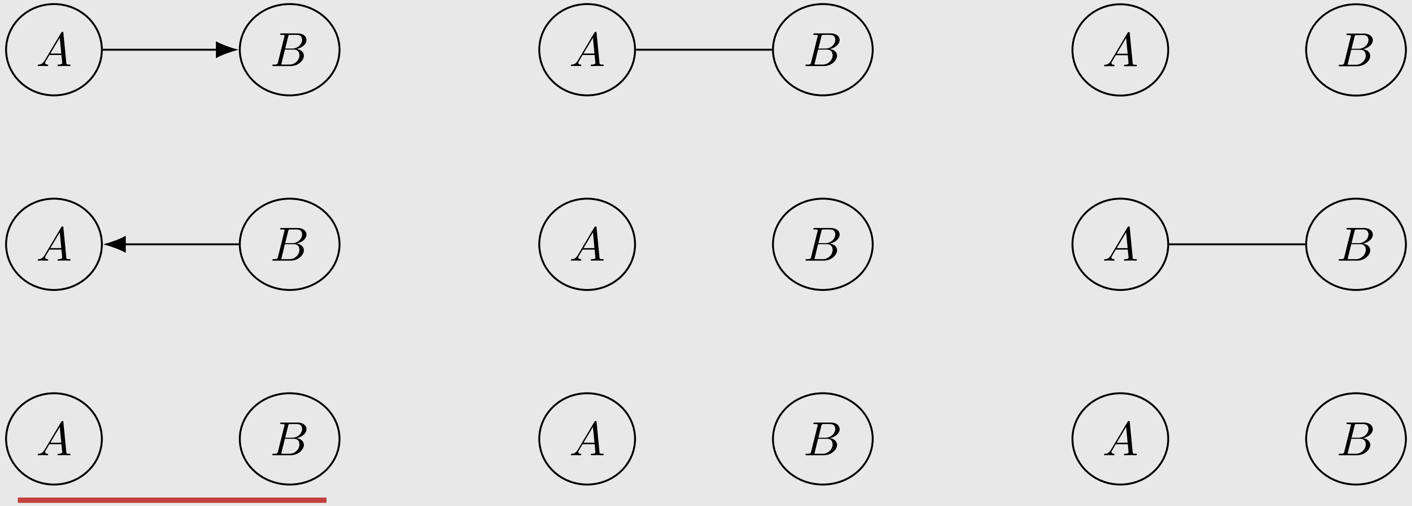

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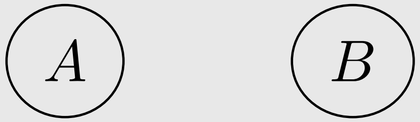

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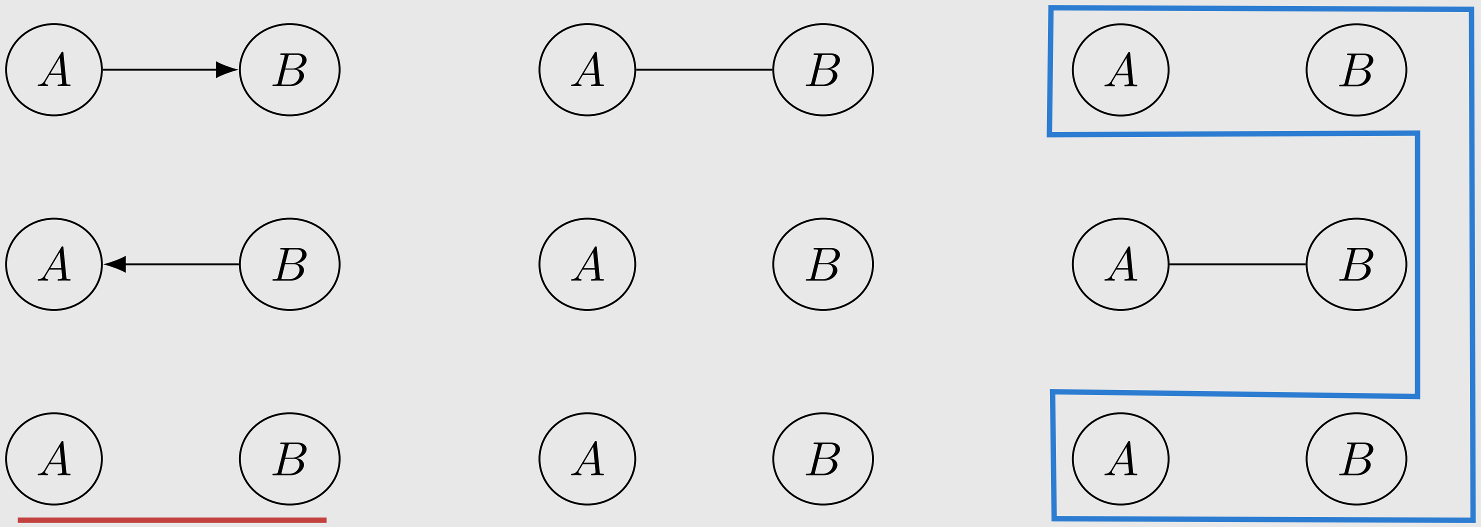

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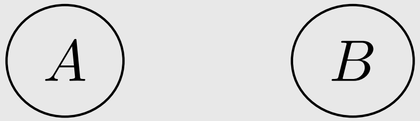

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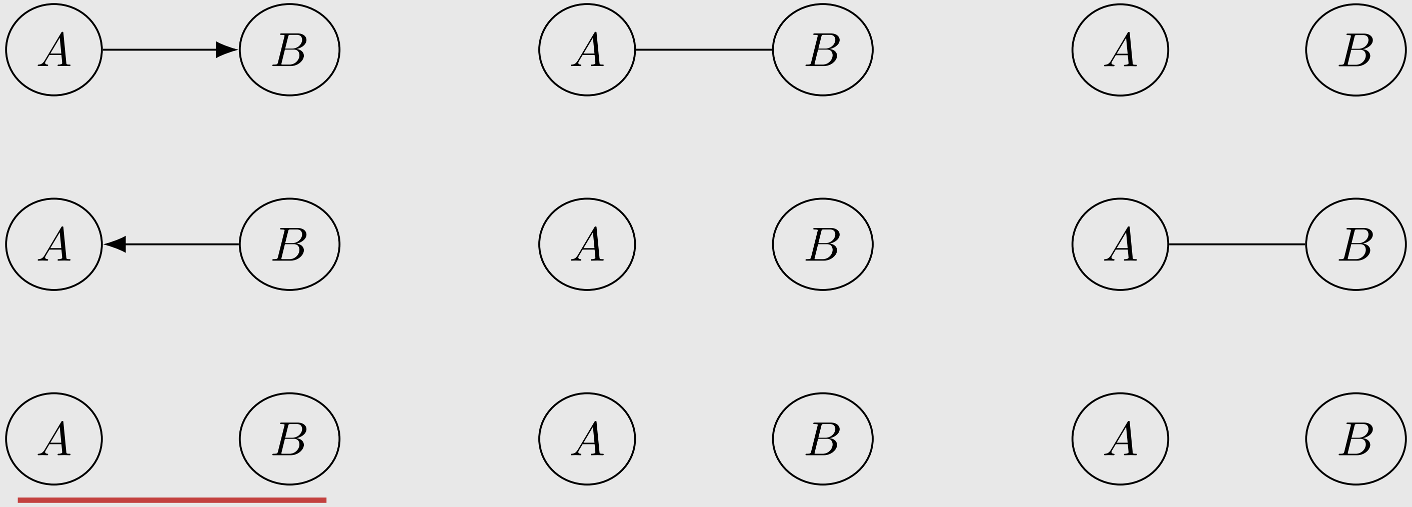

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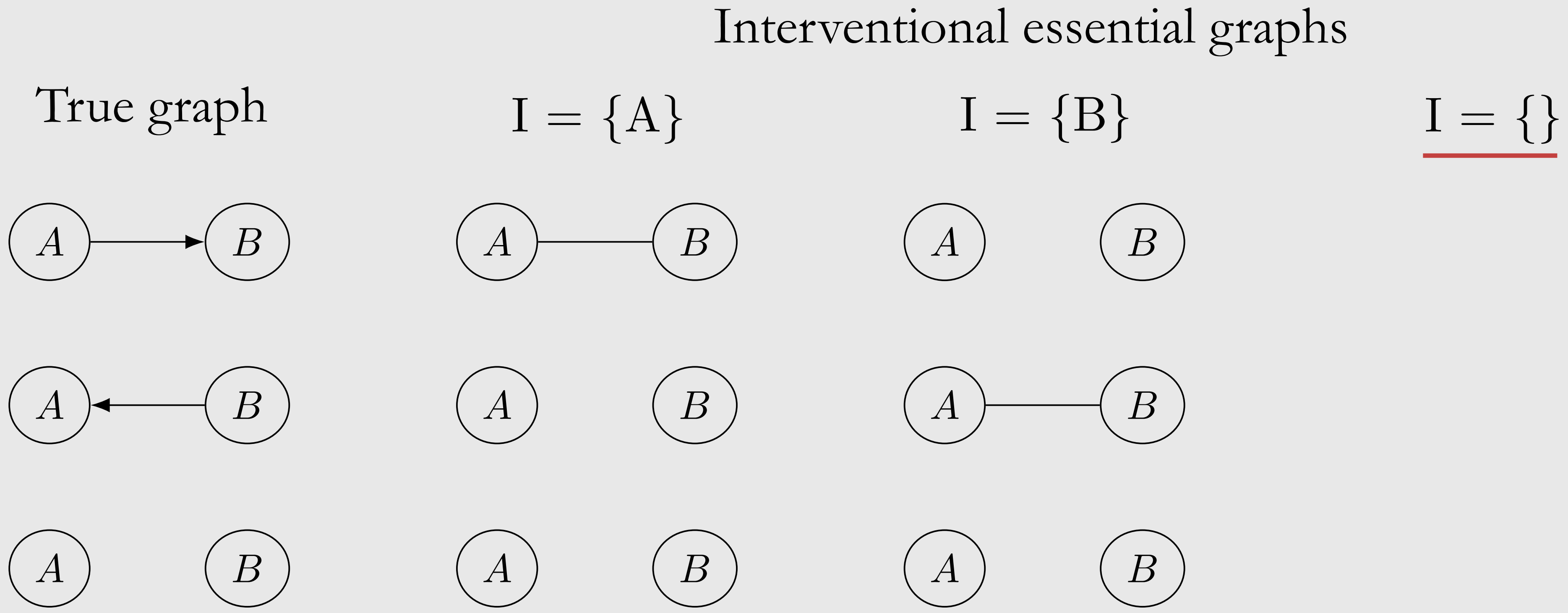

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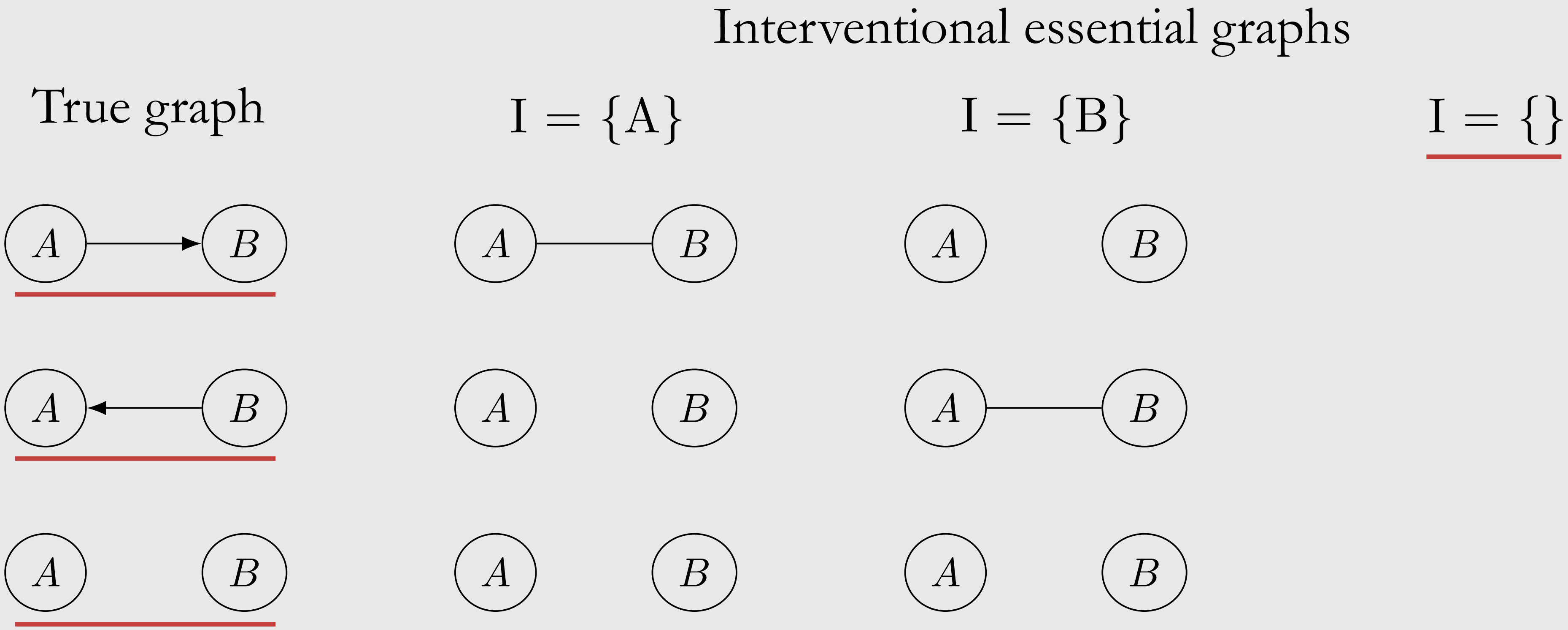

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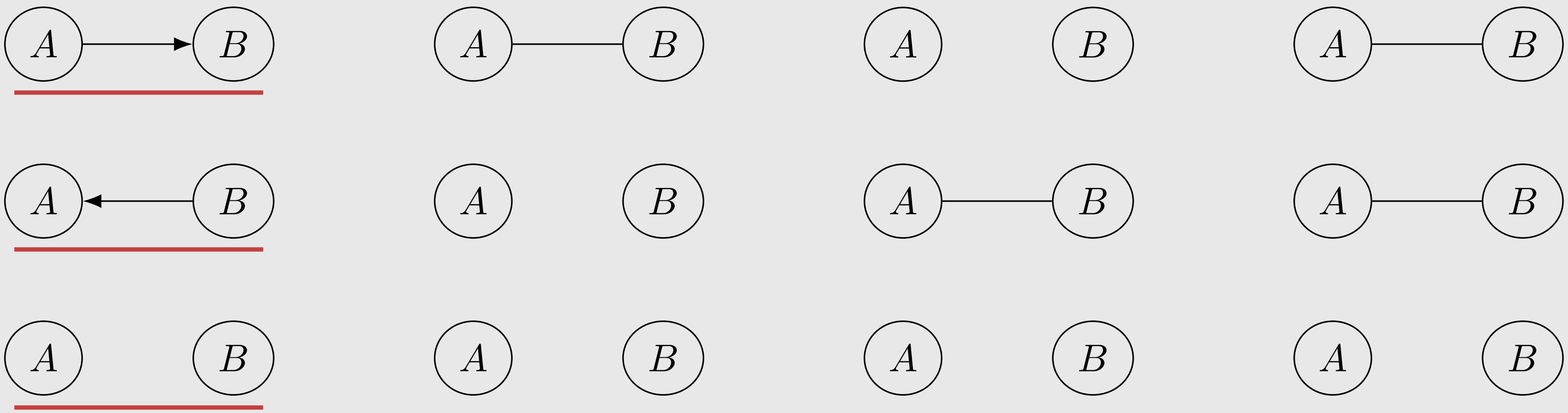

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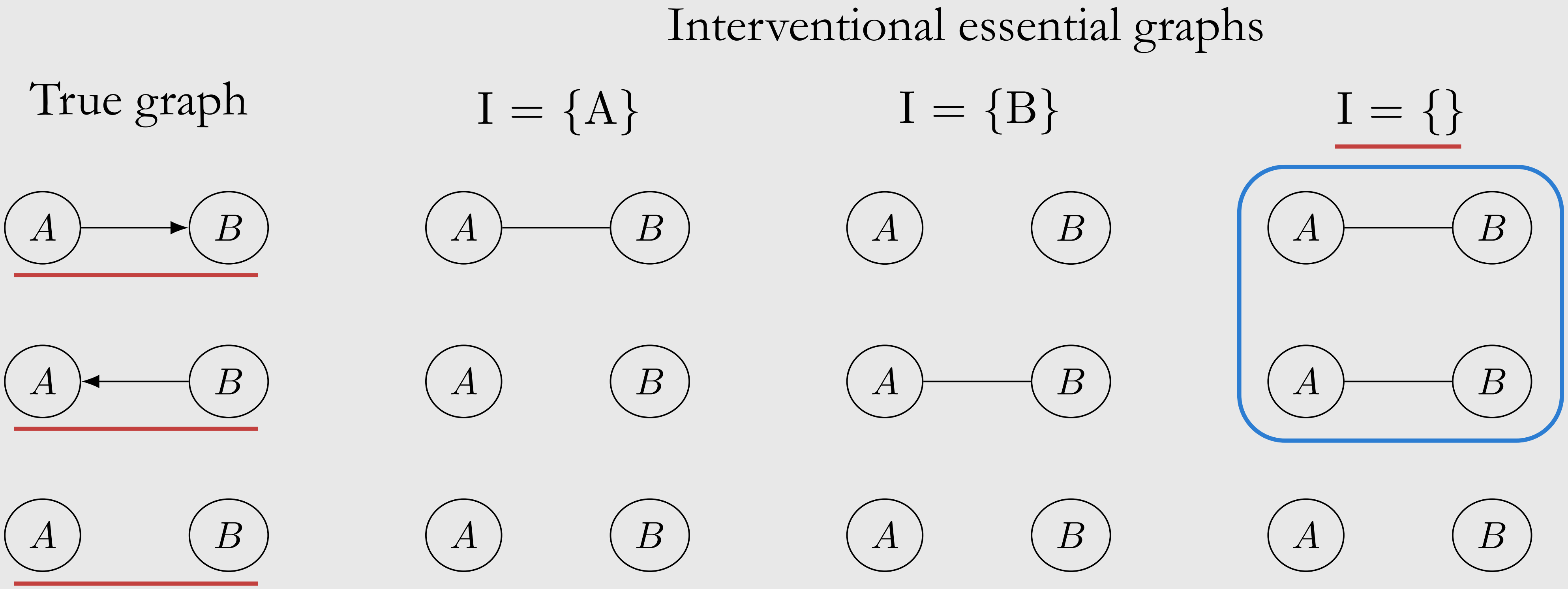

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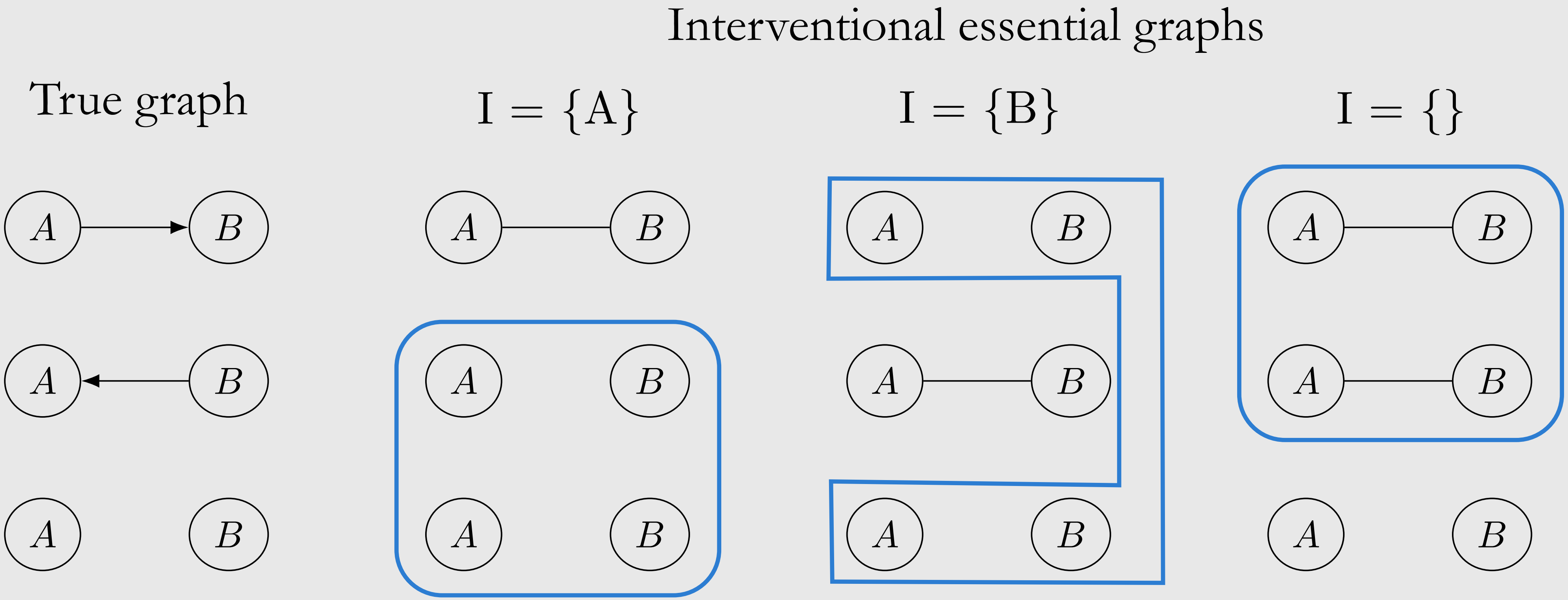
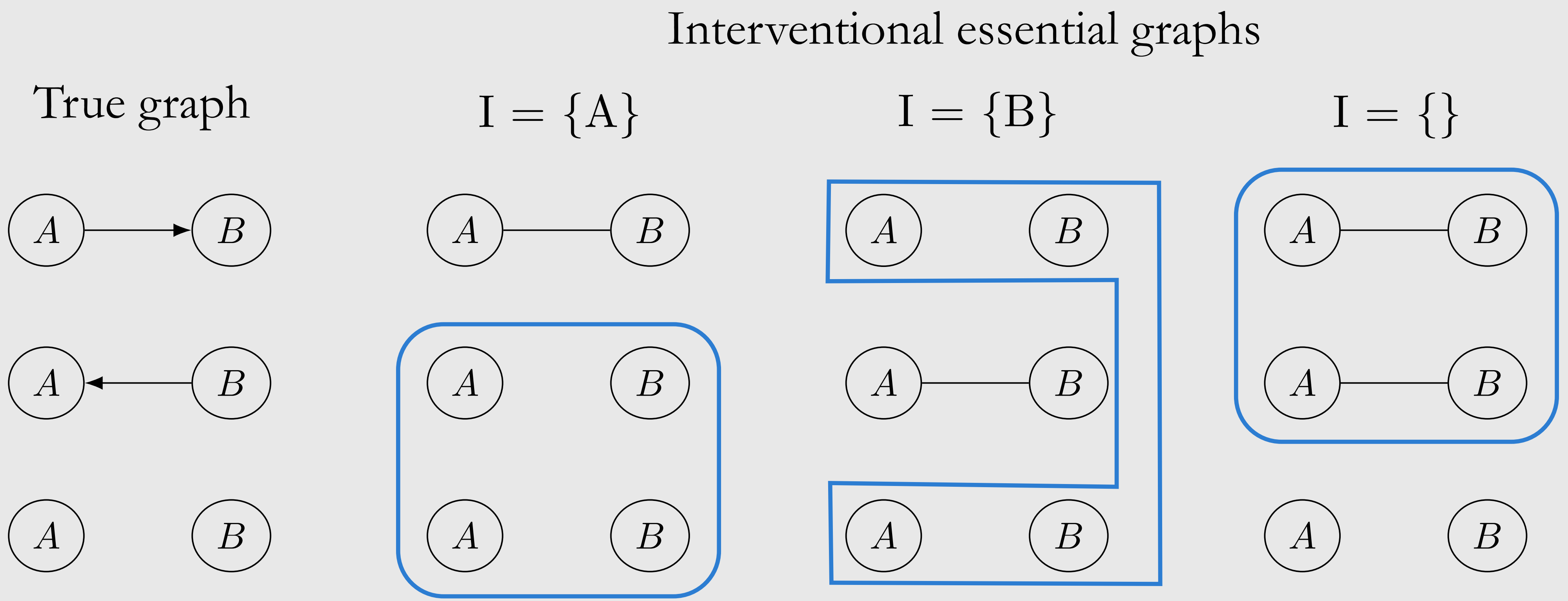


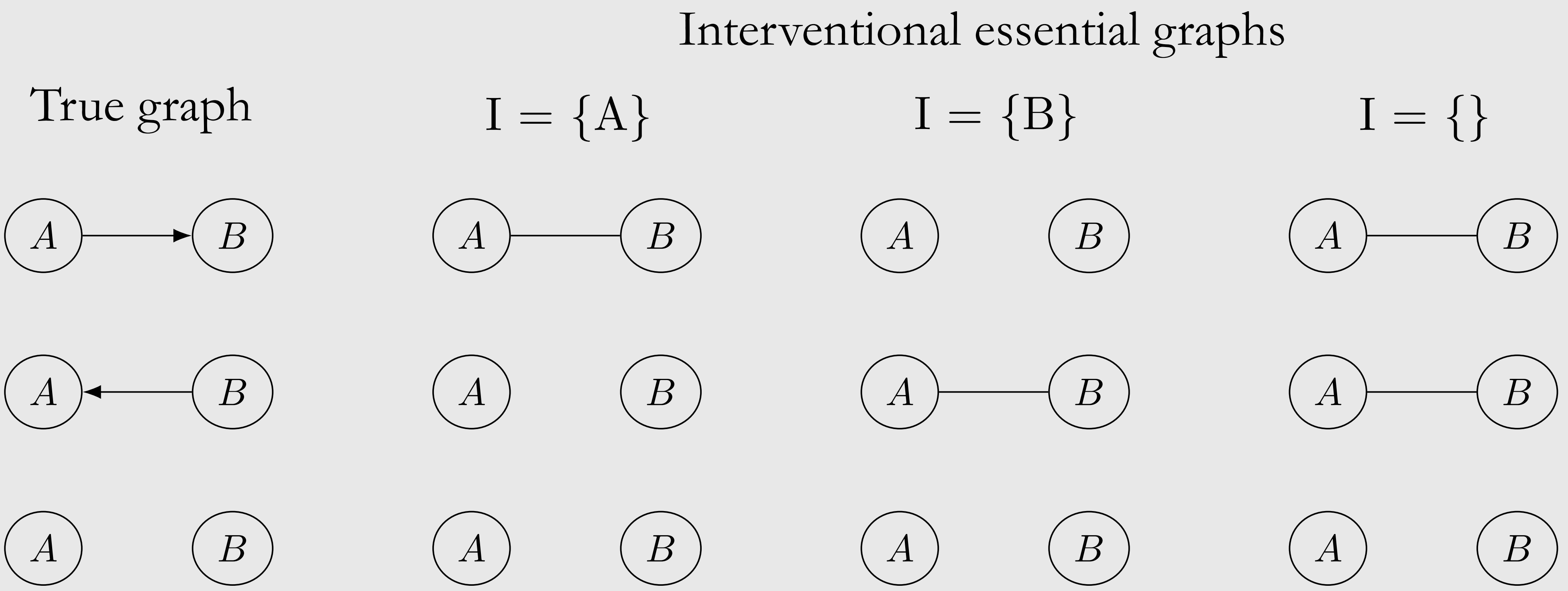
Two Variables: Interventional Essential Graphs

Need more than one intervention to identify the graph



Two Variables: Two Interventions Identify the Graph

Two interventions are sufficient and necessary to identify the graph.



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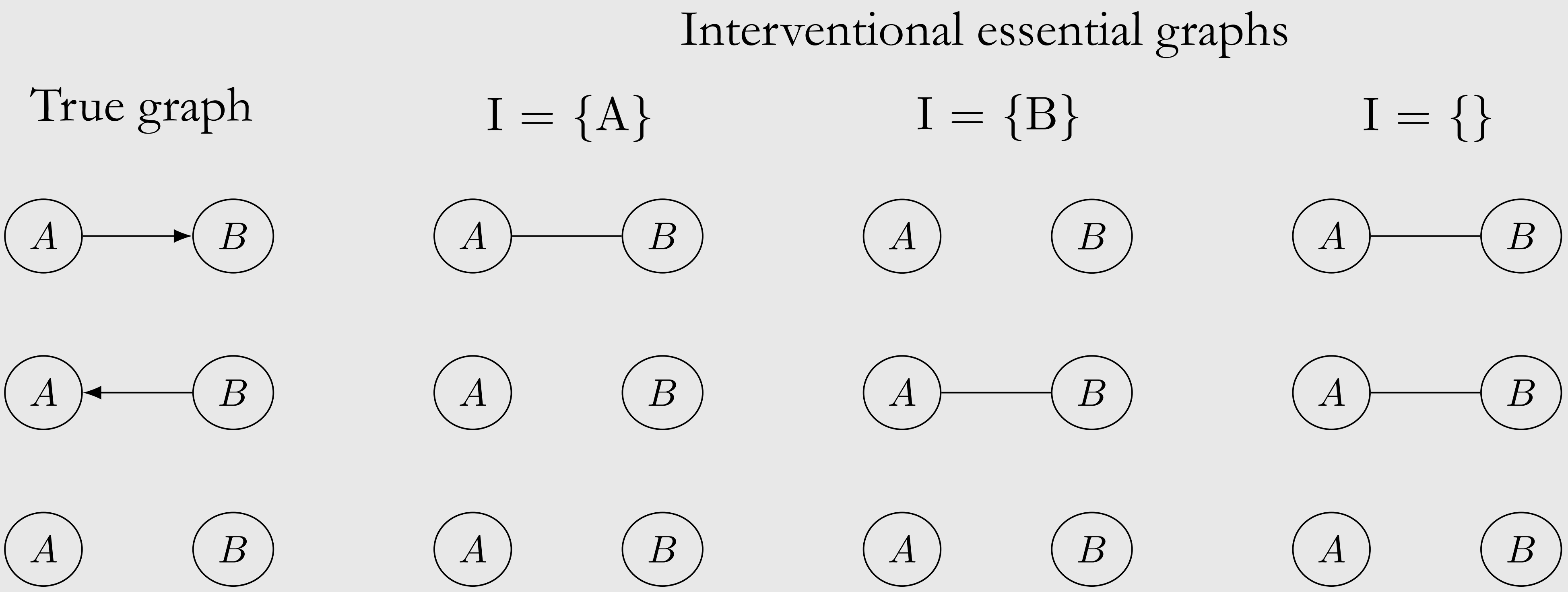

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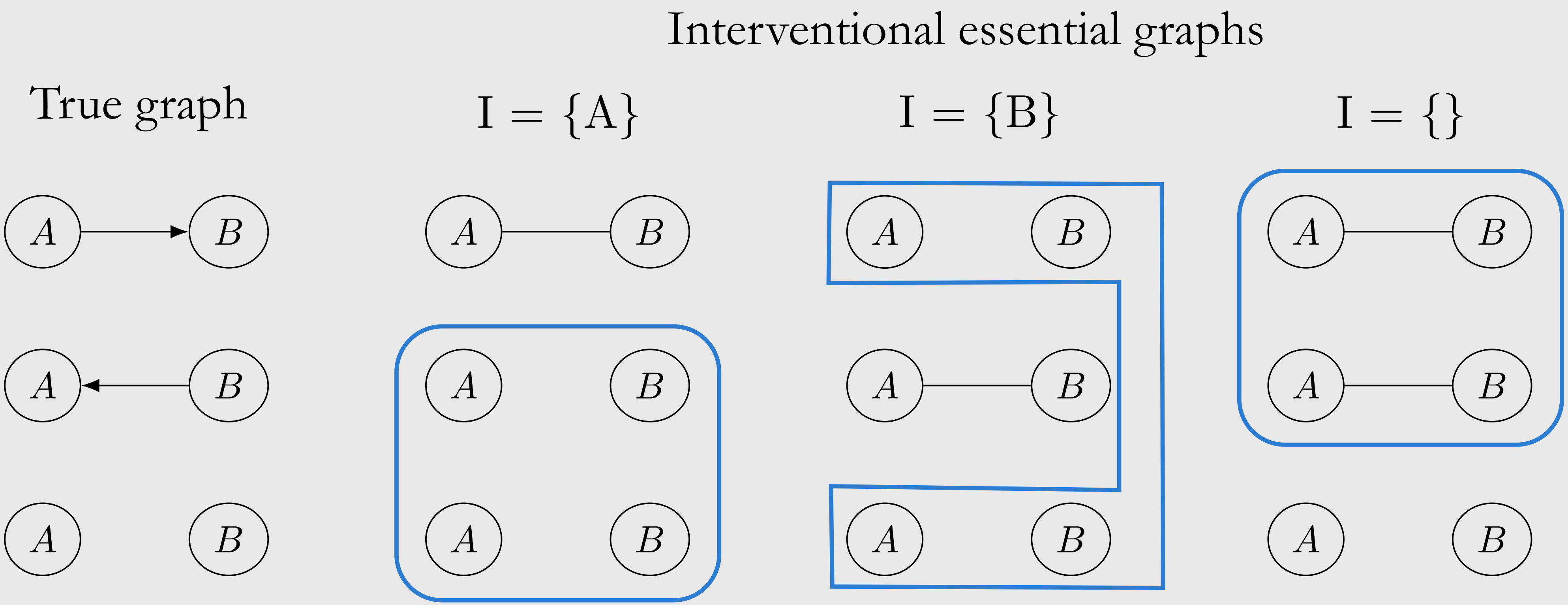

Complete Graphs Are the Worst Case

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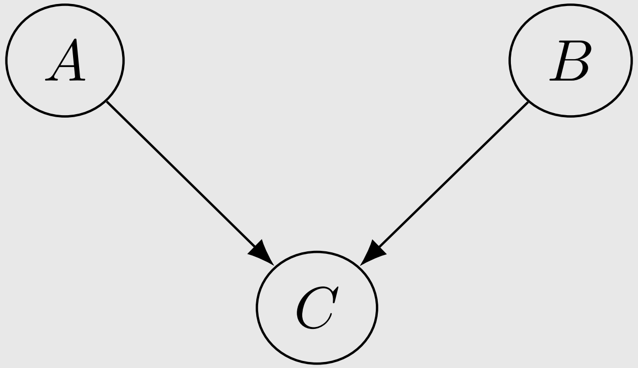
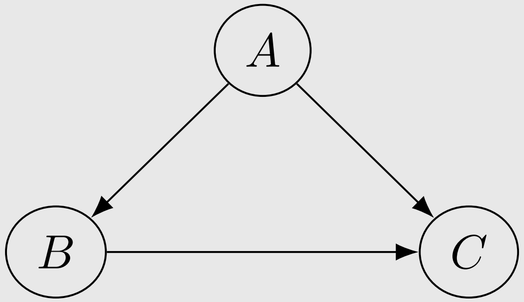
Immorality



Three Variables



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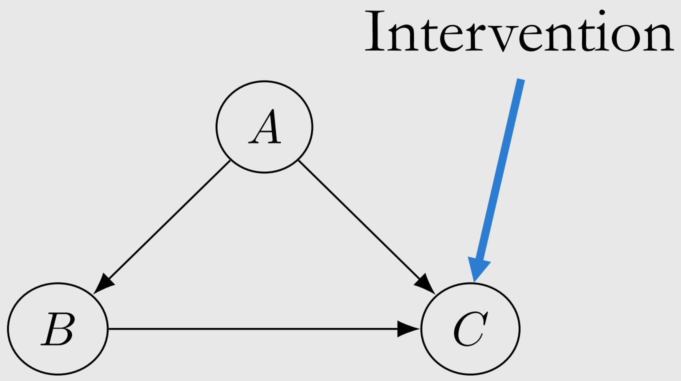

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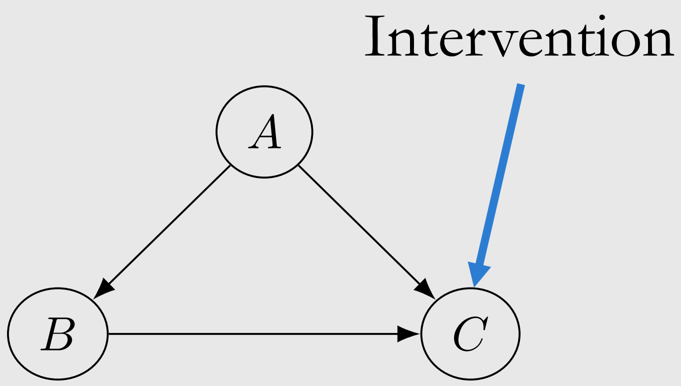

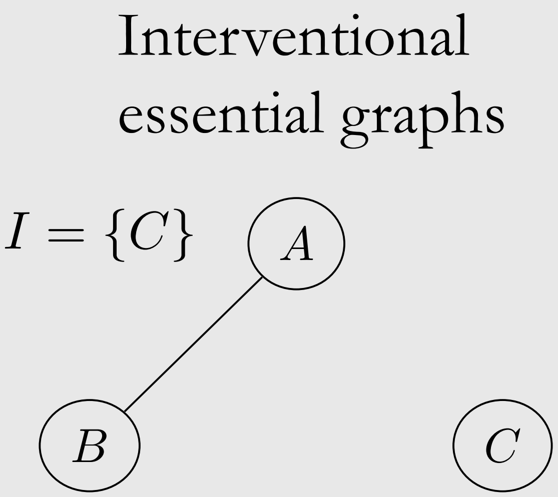

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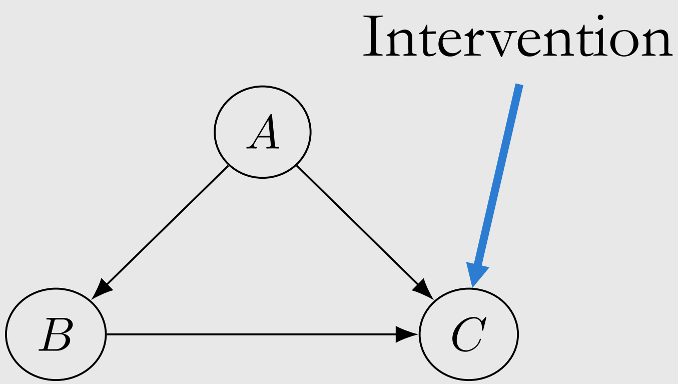
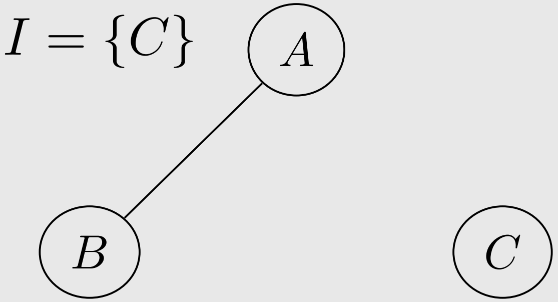


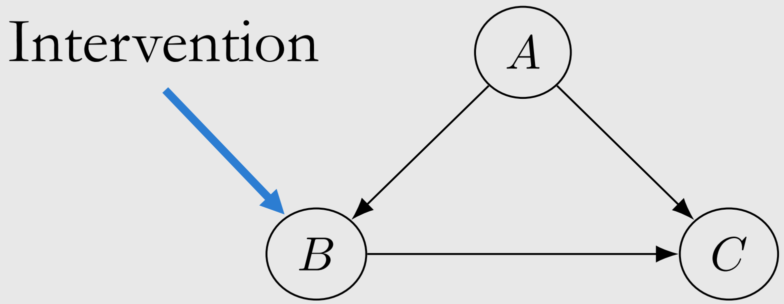
Interventional essential graphs



What we've learned:

- No C → A edge
- No C → B edge
- A and B connected

Three Variables



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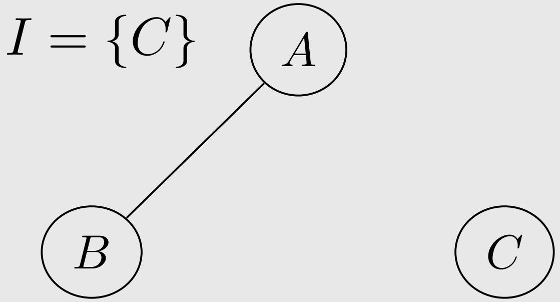

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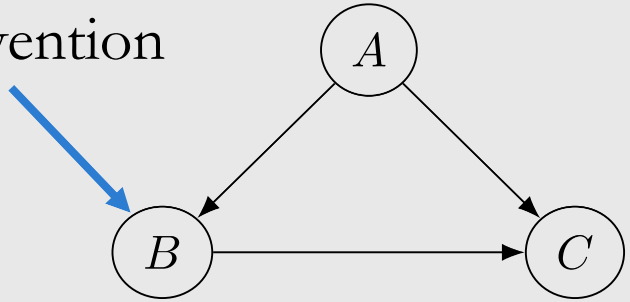
Intervention



Interventional
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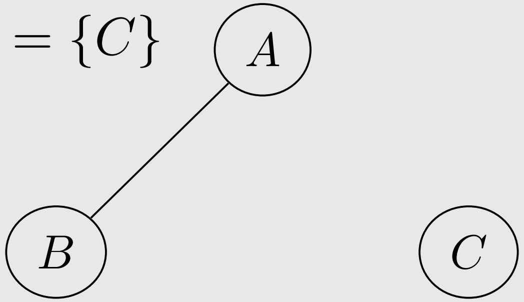

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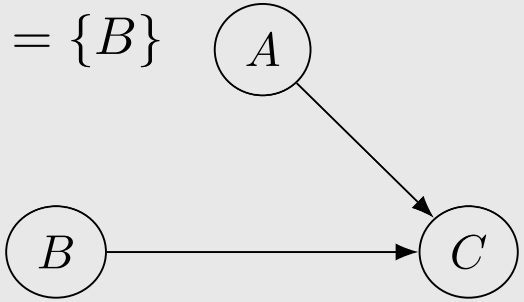

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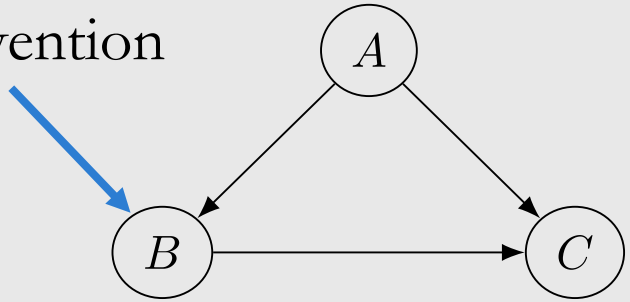
Intervention



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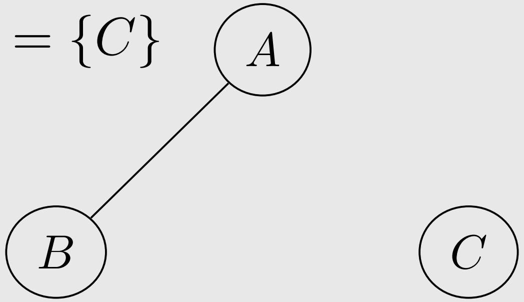

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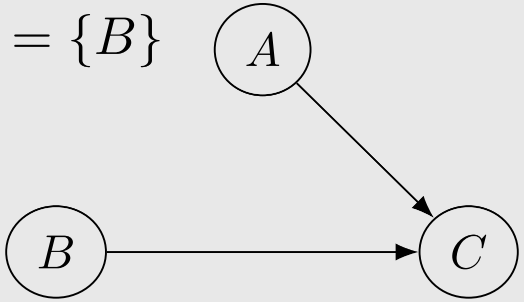

What we've learned:

- No C → A edge
- No C → B edge
- A and B connected
- No B → A edge
- Yes A → C edge
- Yes B → C edge

Single Variable Interventions: n – 1 Are Sufficient for n > 2 (Eberhardt et al., 2006)

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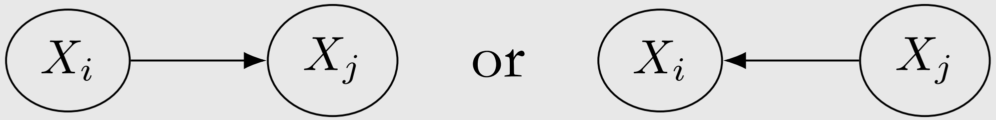

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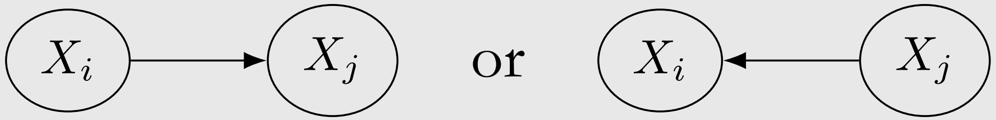

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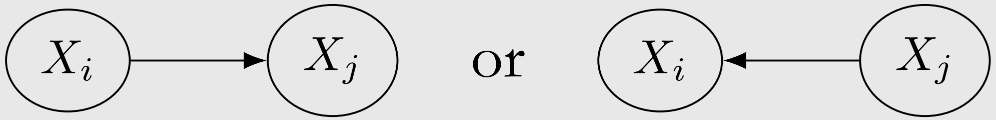

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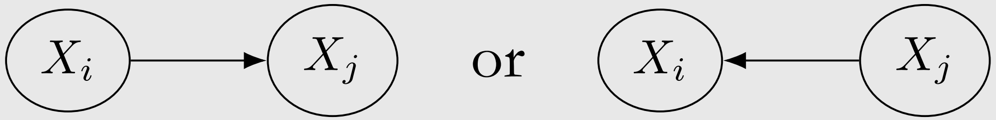

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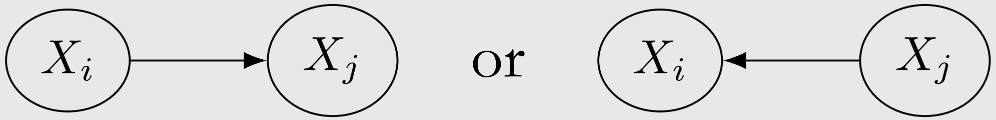

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Note: if you include the empty set (observational), then n are necessary in the worst case (Eberhardt et al., 2006).

Single Variable Interventions: n – 1 Are Necessary in the Worst Case (Complete Graph)

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Adaptivity doesn't help in the worst case (Eberhardt et al., 2006).

Questions (where the goal is to identify the graph):

1. Show that n interventions are necessary in the worse case in the three-variable setting when you use the observational data (null intervention) as one of the interventions.
2. What is the minimum number of interventions necessary in the worst case when n = 3?
3. What is the minimum number of interventions necessary in the worst case when n = 2?

Structural Interventions

Single-Node Interventions

Multi-Node Interventions

Parametric Interventions

Interventional Markov Equivalence

Miscellaneous Other Settings

Multiple-Node Interventions

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Eberhardt et al. (2005) found that $\lfloor \log_2(n) \rfloor + 1$ multi-node interventions are necessary for worst case graphs (complete graph), but what about for non-complete graphs?

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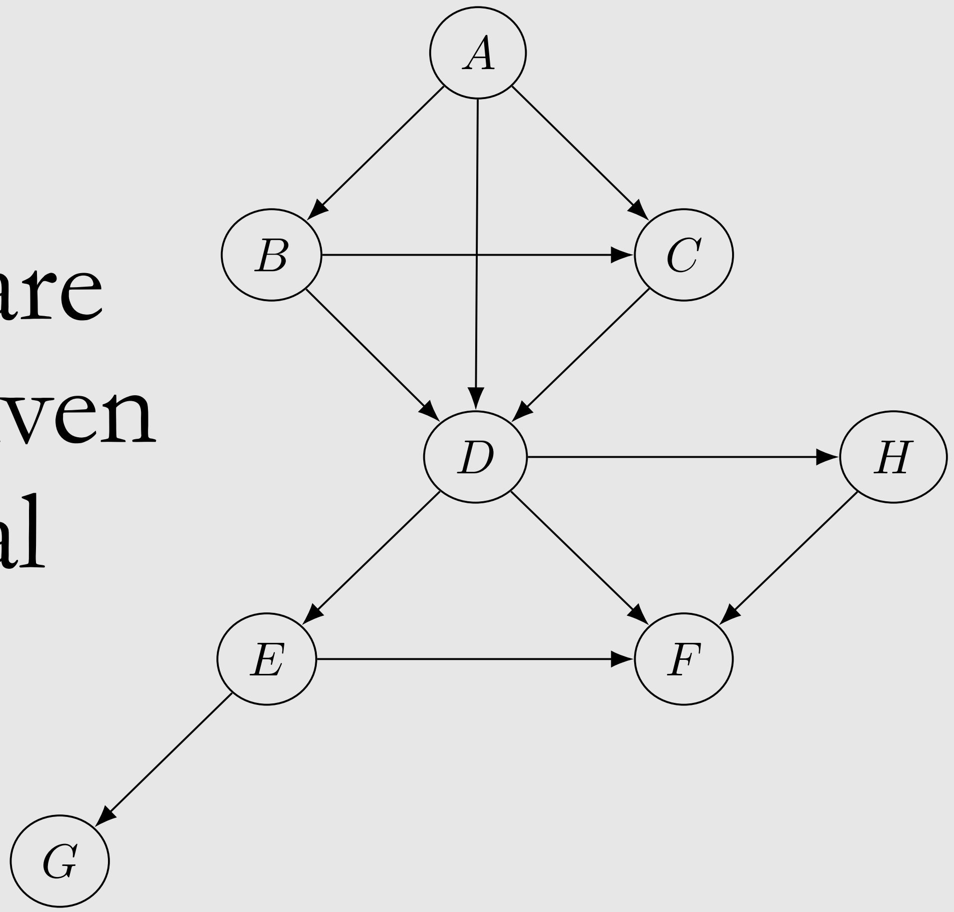
Theorem: $\lceil \log_2(c) \rceil$ multi-node interventions are sufficient and necessary in the worst case, where c is the size of the largest clique (conjectured by Eberhardt (2008) and proven by Hauser & Bühlmann (2014)).

Question:
In this graph, how many multi-node interventions are sufficient and necessary given that you know the essential graph?



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Structural vs. Parametric Interventions

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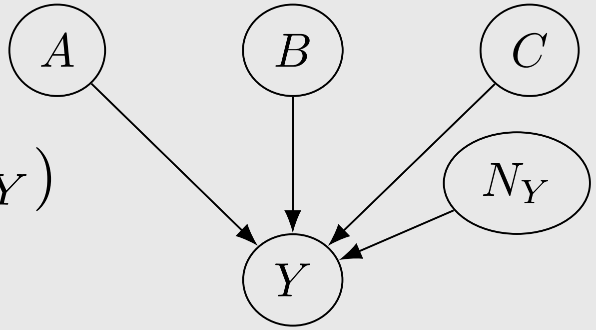
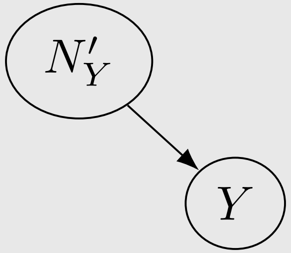
$$Y := f_\theta(A, B, C, N_Y)$$



Structural vs. Parametric Interventions

Structural:

$$Y := N'_Y$$



Structural vs. Parametric Interventions

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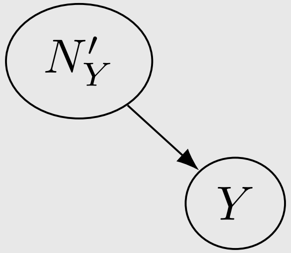
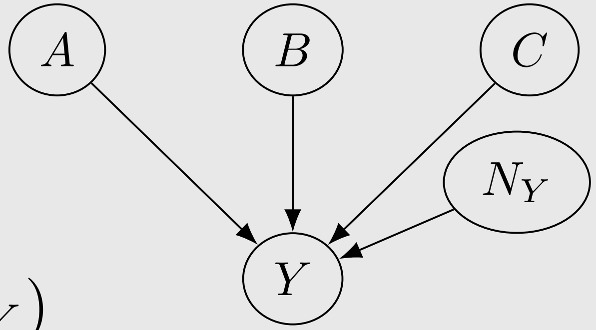


Parametric:



$$Y := f_\theta(A, B, C, N_Y)$$

Structural vs. Parametric Interventions

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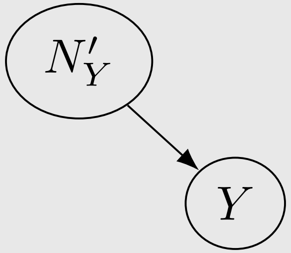

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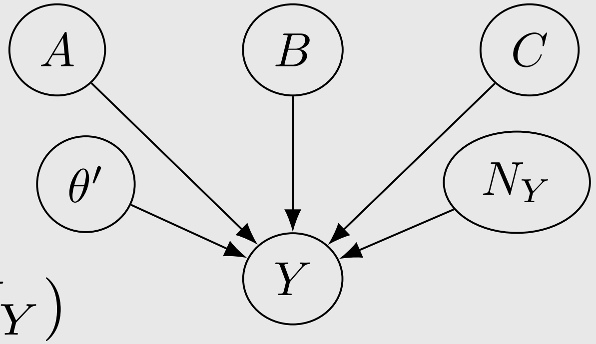

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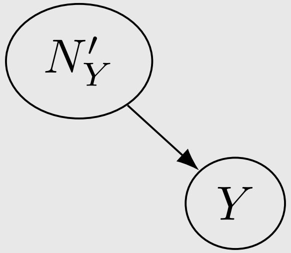

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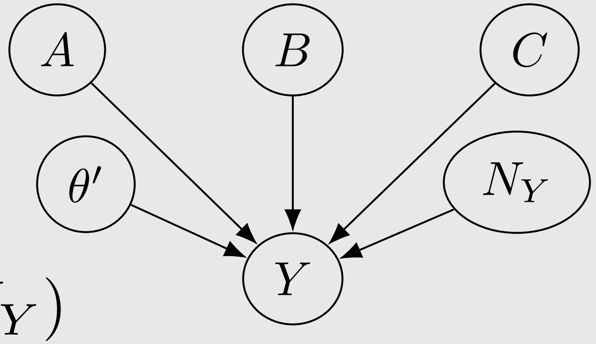

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Other names: hard vs. soft, perfect vs. imperfect, etc.

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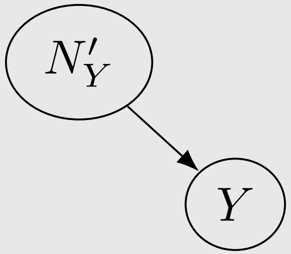

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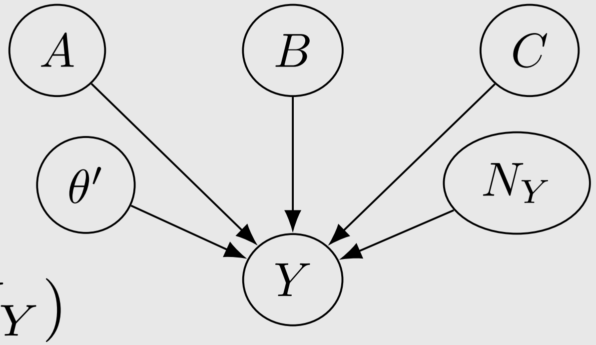

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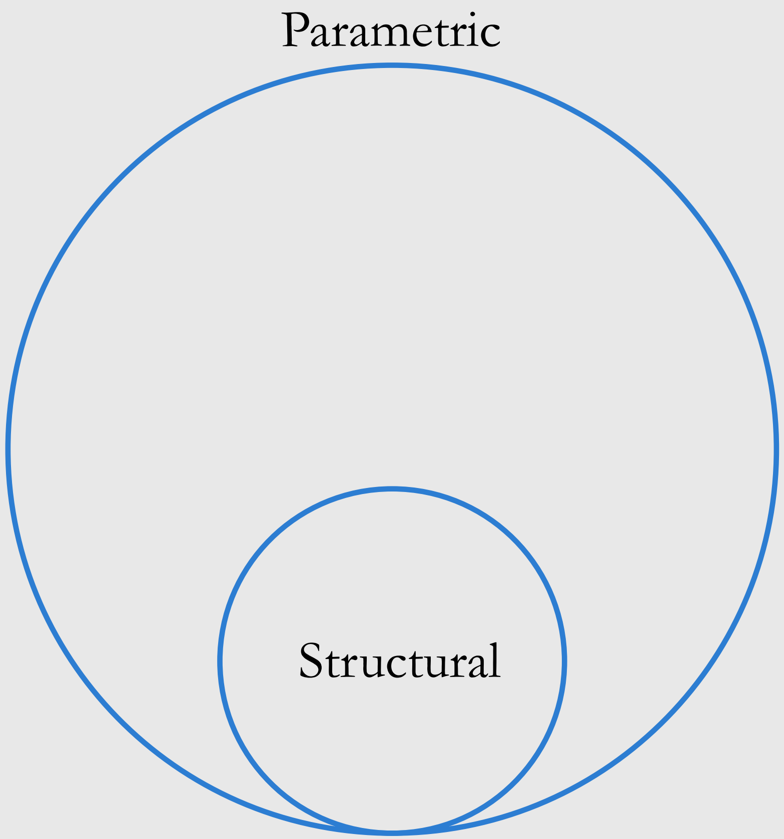

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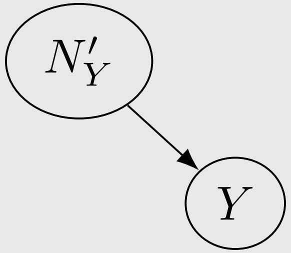

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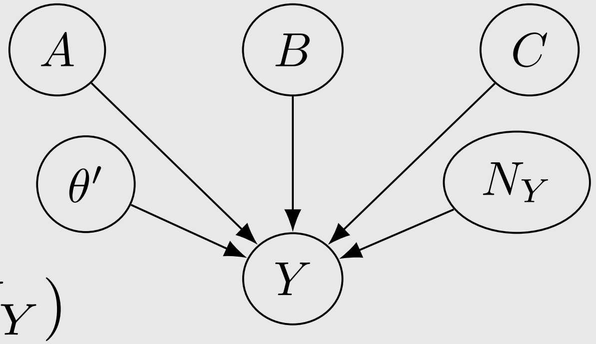

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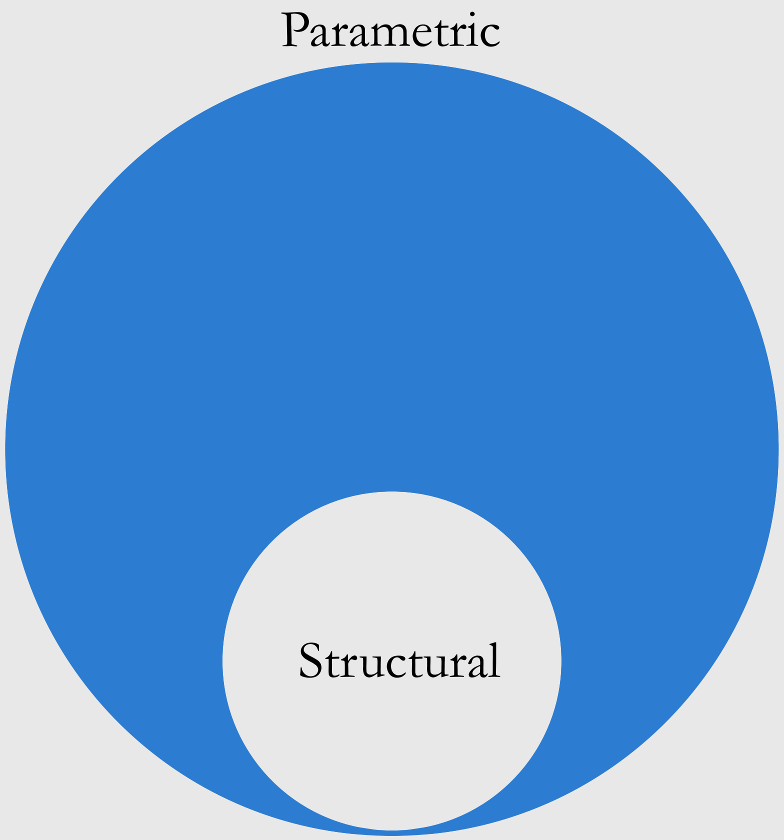

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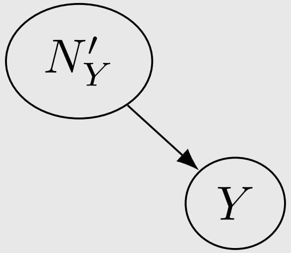

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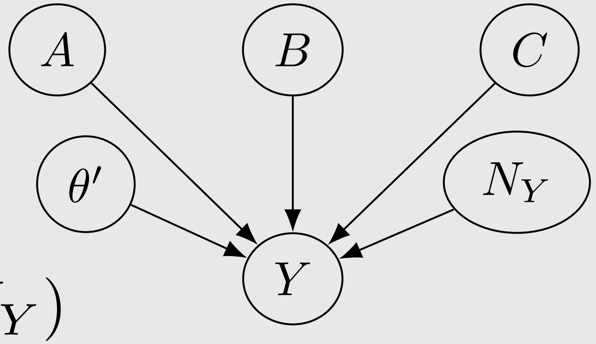

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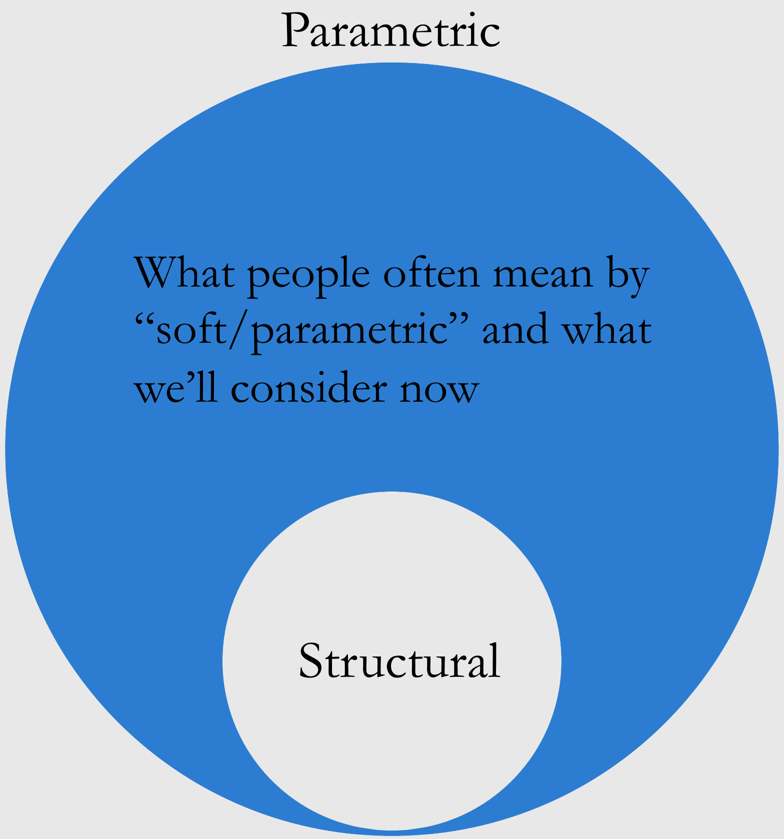

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How about the in-between? How much of the graph can we identify with fewer interventions?

How much of the graph can we identify with a given set of interventions?

Structural Interventions

Single-Node Interventions

Multi-Node Interventions

Parametric Interventions

Interventional Markov Equivalence

Miscellaneous Other Settings

Interventions Introduce Immoralities: Single-Node

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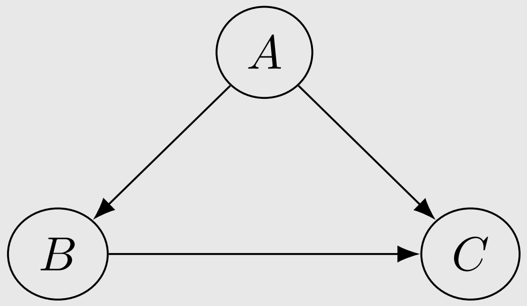
True Graph



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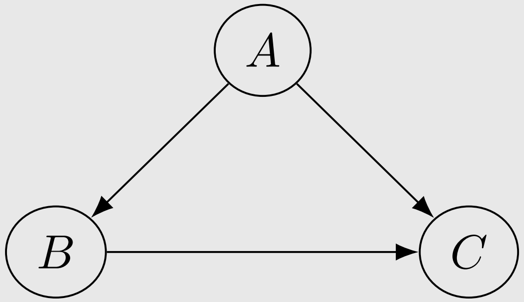
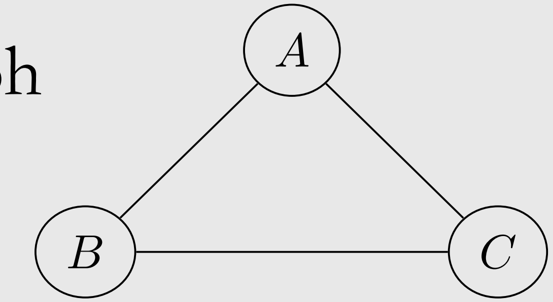


Essential Graph



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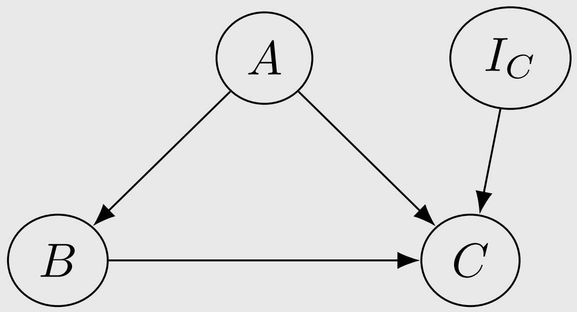
Interventional Graph

True Graph



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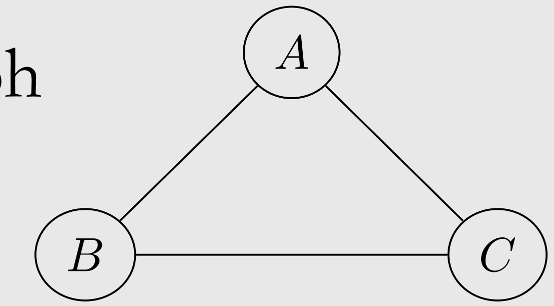

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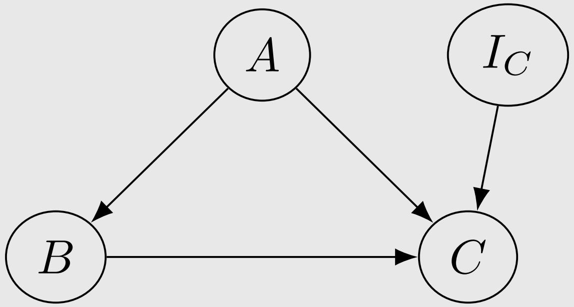

Preserves structure, unlike structural interventions

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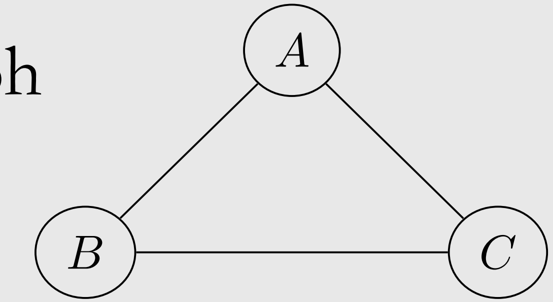

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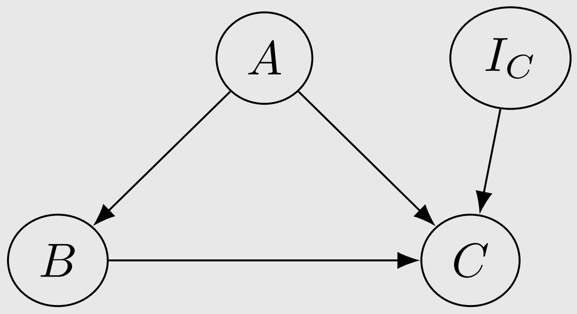

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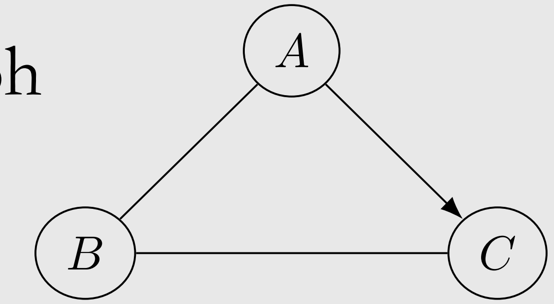

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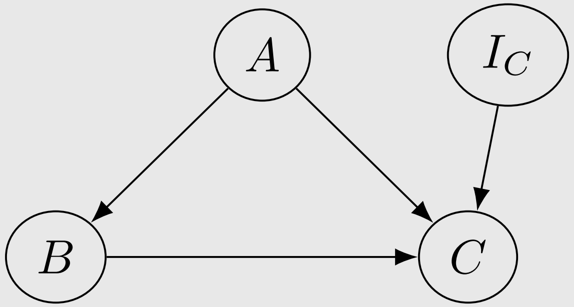

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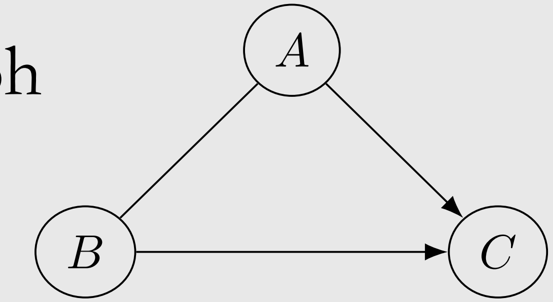

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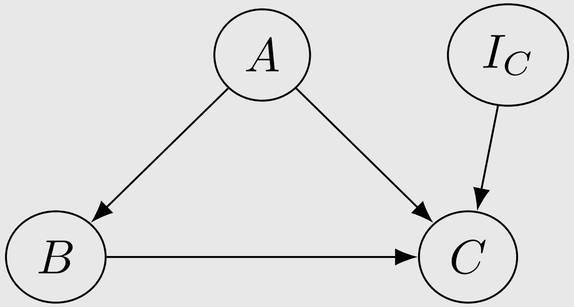

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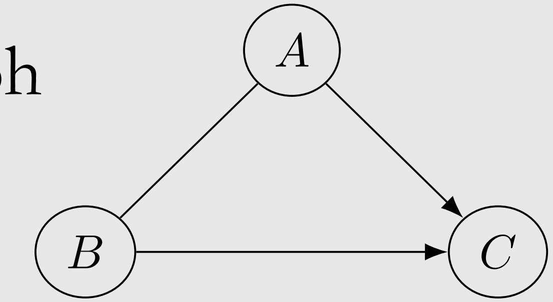
Interventional Essential Graph



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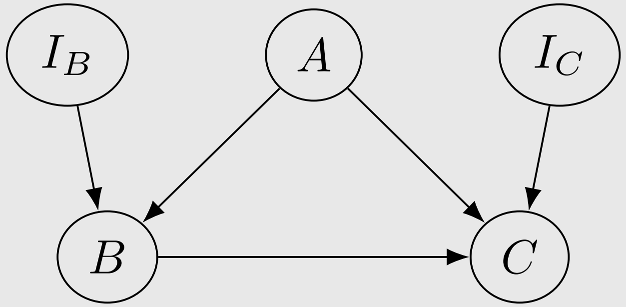

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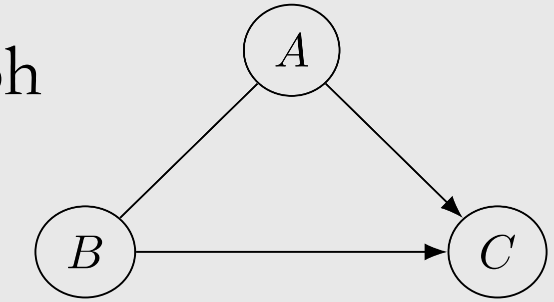

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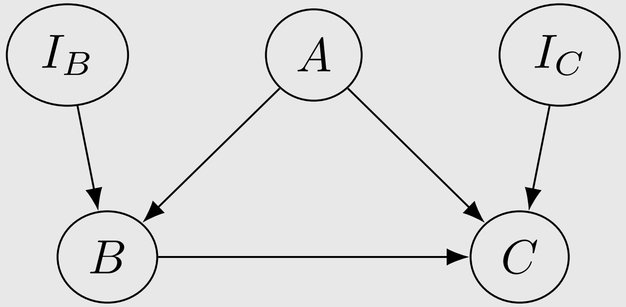

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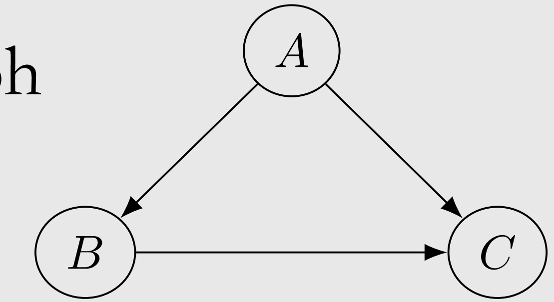

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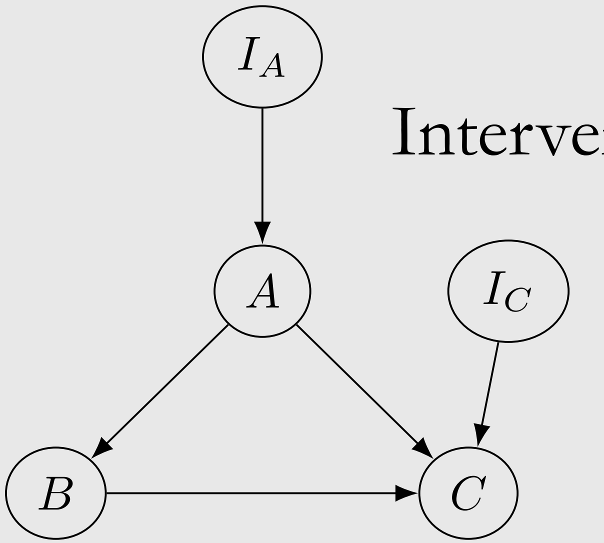

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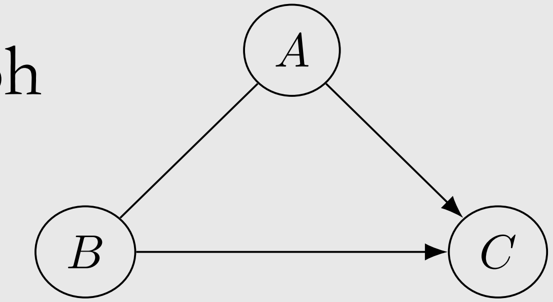

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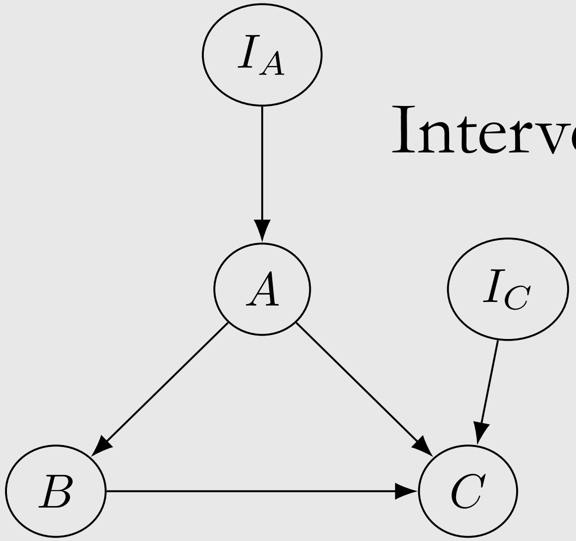

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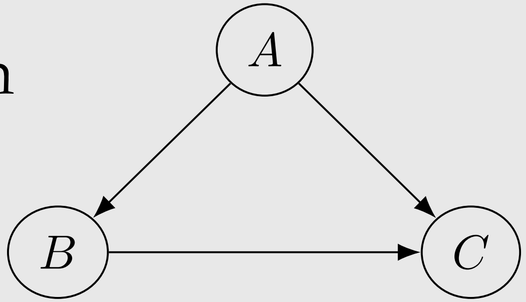

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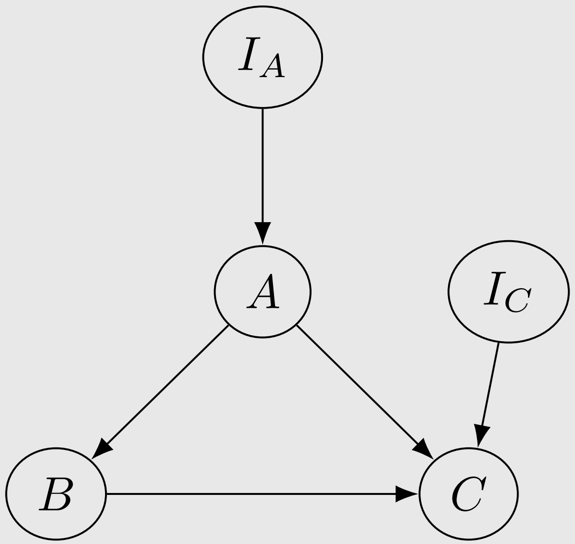
Interventional Graph: Multi-Node Interventions

Two Single-Node Interventions



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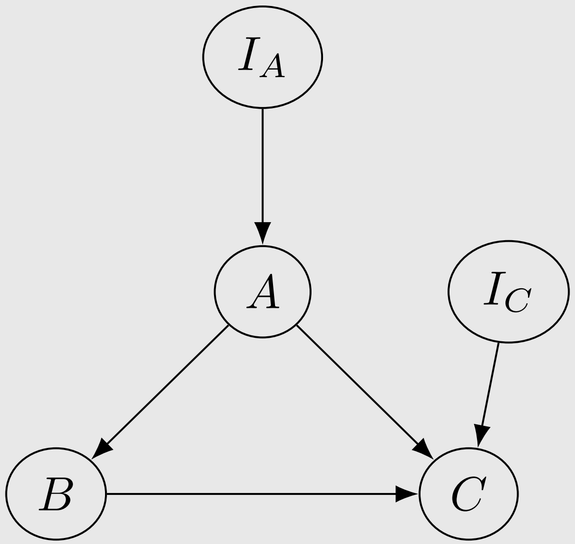

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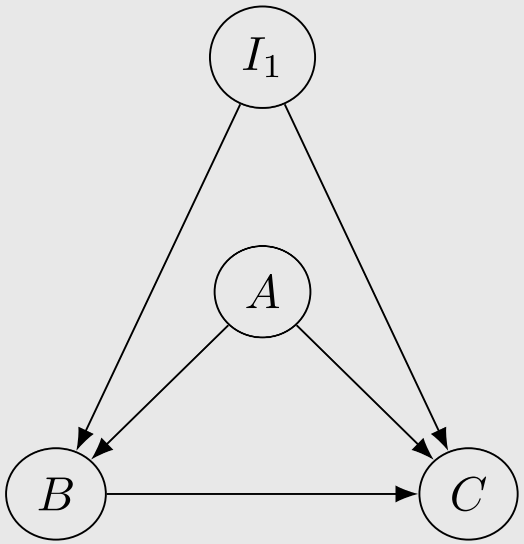

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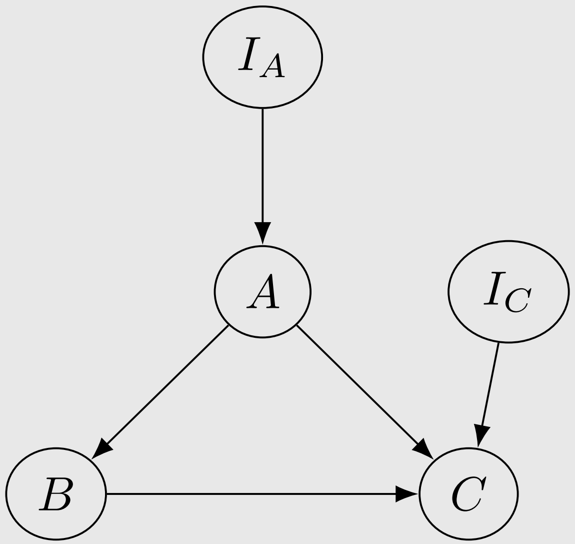

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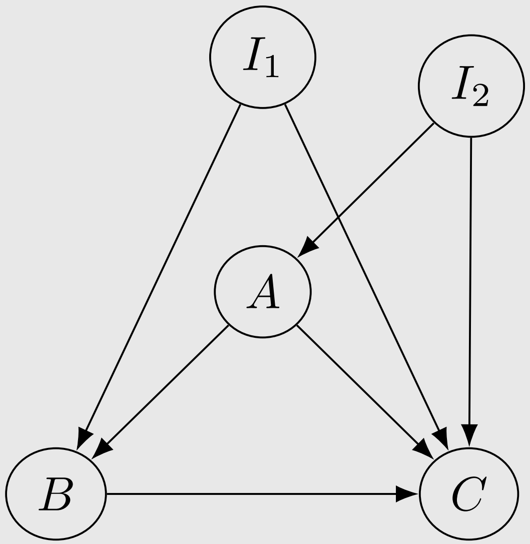

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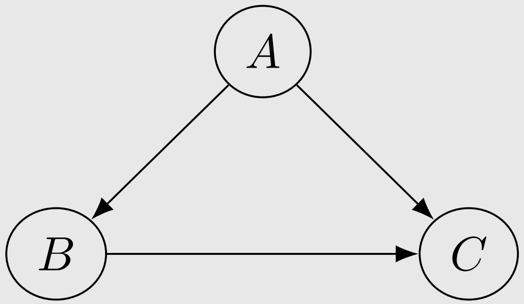

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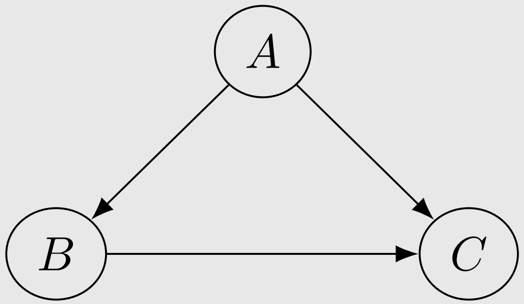

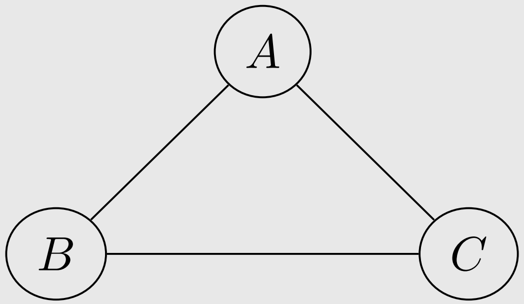

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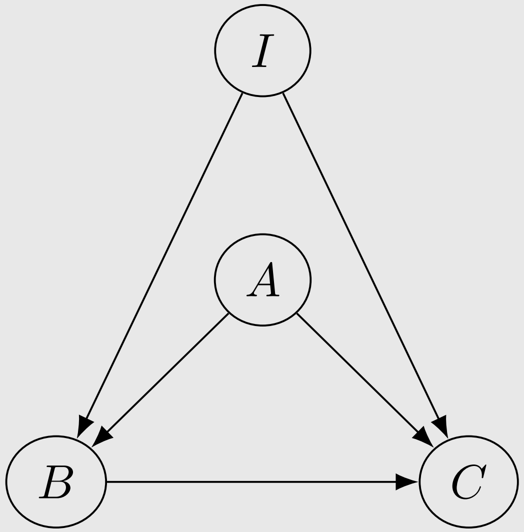

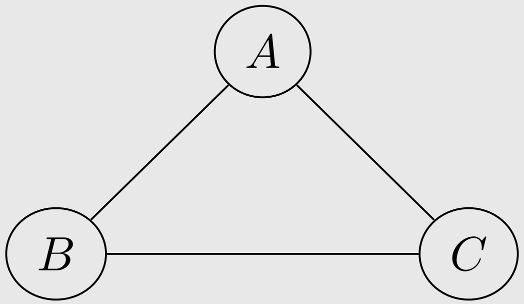

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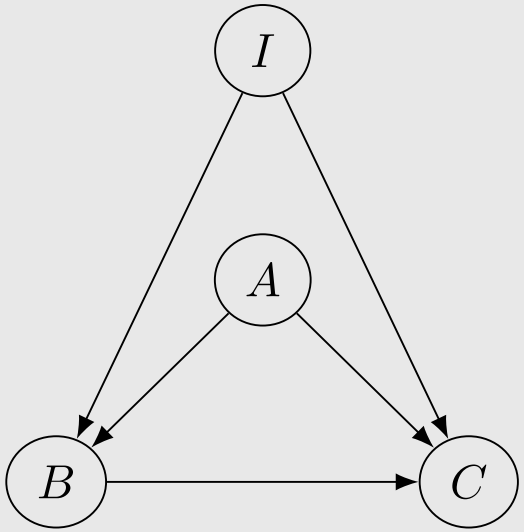

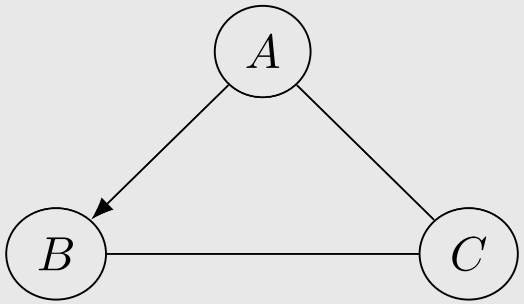

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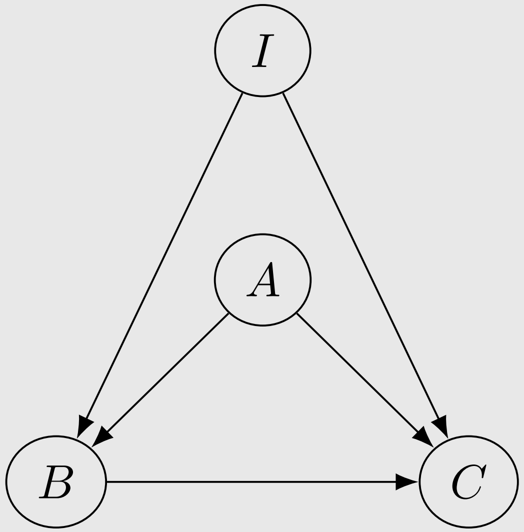

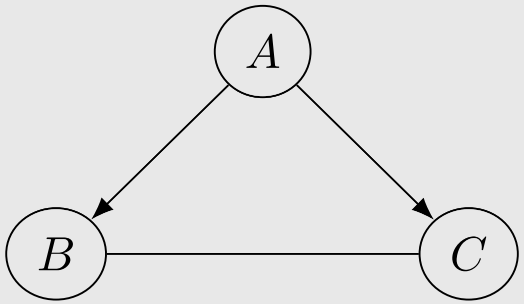

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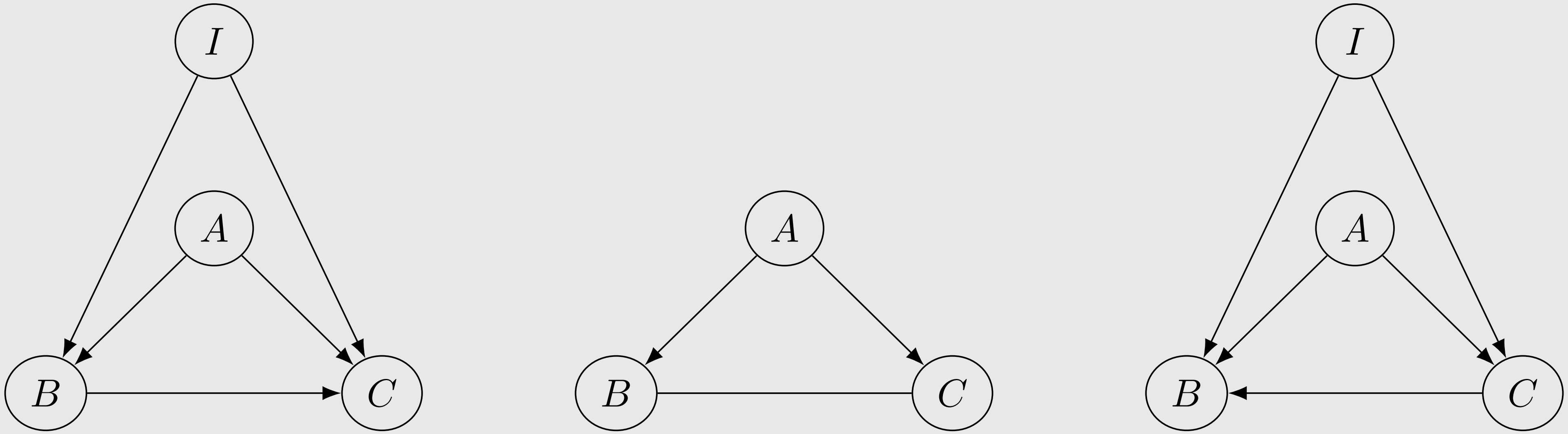

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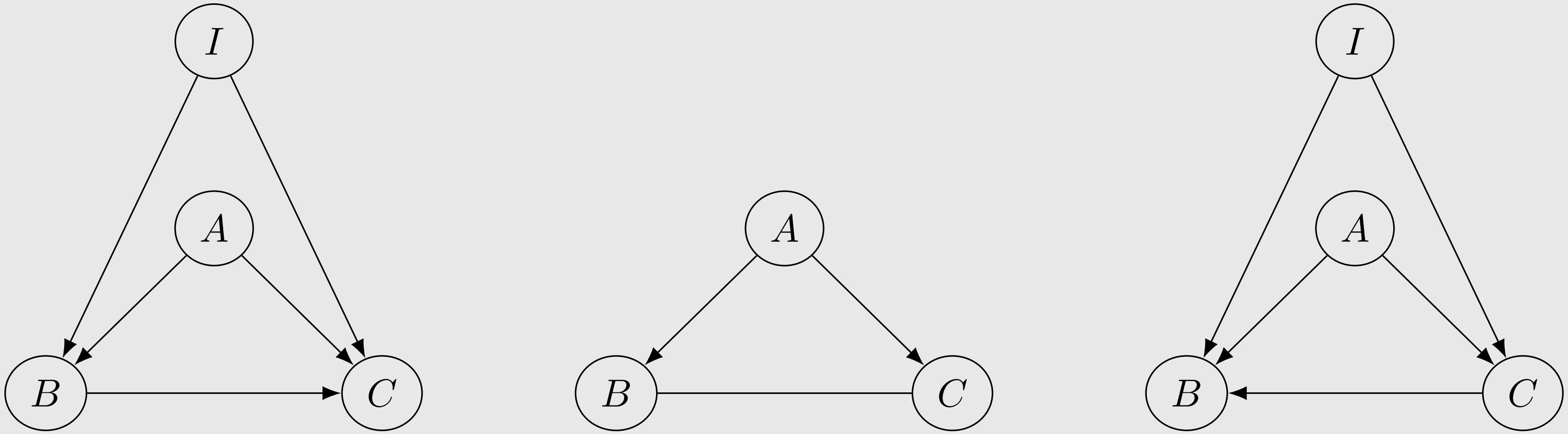
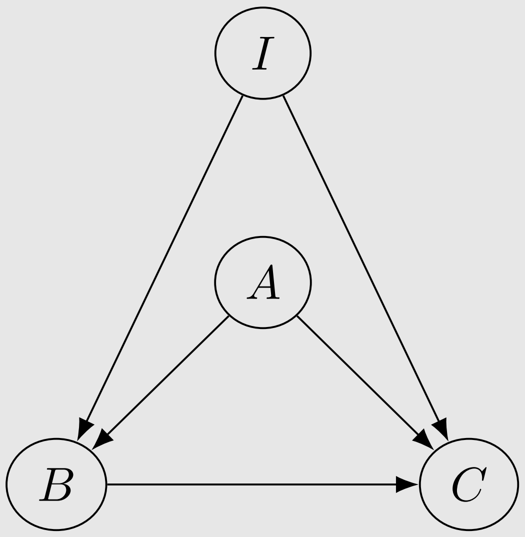
Structural/perfect analog from Hauser & Bühlmann (2012)



Questions

1. How many parametric single-node interventions are necessary and sufficient for identifying a directed acyclic graph?
2. What is the essential graph of the graph on the right (ignoring the intervention node)?
3. What is the interventional essential graph?
4. What is a graph that this graph is interventionally Markov equivalent to?



Structural Interventions

Single-Node Interventions

Multi-Node Interventions

Parametric Interventions

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Miscellaneous Other Settings

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- Randomized algorithms
  - Only need O(log log n) interventions with high probability (Hu et al., 2014)
  - Better than O(log c) from Eberhardt (2008) and Hauser & Bühlmann (2014)
- Intervene on at most k variables per intervention: Shanmugam et al. (2015)
  - Worst case: $\sim \frac{n}{2k}$
  - Randomized: $O\left(\frac{n}{k}\log\log k\right)$
- Only k interventions: Ghassami et al. (2018)
- Unobserved confounding: Kocaoglu et al. (2017)